Graph Refinement based Airway Extraction

Using Mean-field and Graph Neural Networks

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Outline

1. Airway Diseases
2. Understanding the Data
3. Methods
   - Existing Methods
   - Graph Refinement Model
   - Mean-Field Networks
   - Graph Neural Networks
4. Experiments

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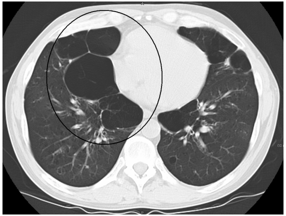

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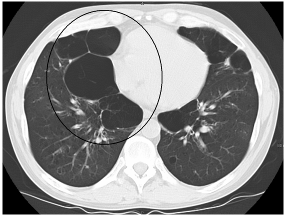

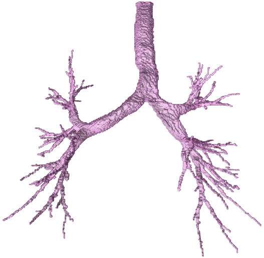

Existing Diagnostics are Rudimentary & Tedious

- Lung Function Tests
  - \+ Simple and inexpensive
  - – Patient dependent
  - – Low reproducibility
  - – Mild cases go unnoticed

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  + Simple and inexpensive
  – Patient dependent
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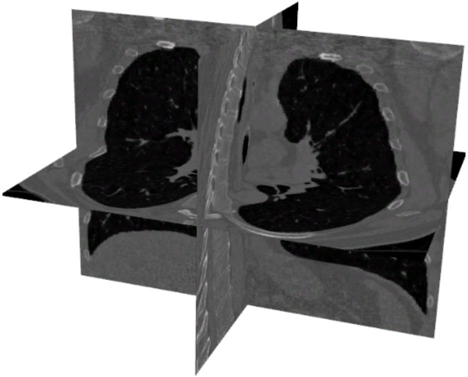
- 3D CT Scans
  + Provide more information
  – Arduous to read the data; even for experts
  – Low inter-observer agreement



Objective

Automatic Airway Segmentation and obtain **useful** COPD biomarkers

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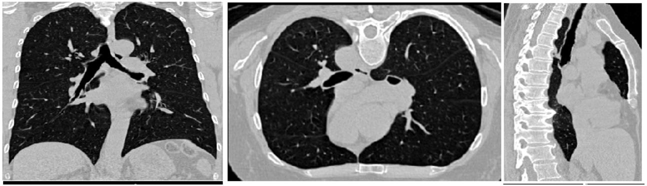
Primary Data from Danish Lung Cancer Study

- Danish Lung Cancer Screening Trial [1]
- Low-dose CT
- $> 10,000$ scans
- Age 50-70 years.
- Smoker or former smoker ($>$ 20 pack years)
- 32/10,000 have segmentations verified by an expert user!

[1] Pedersen, J. H., et.al : The Danish randomized lung cancer CT screening trialoverall design and results of the prevalence round. Journal of Thoracic Oncology, (2009)

CT Images are noisy, low contrast & low-res.



- Volume resolution $\sim$ 300x250x275
- Voxels $\sim$ 0.75mmx0.75mmx1mm
- Challenges
  - Acquisition noise
  - Inter-patient variability
  - Several "interfering" structures
  - Labels/Annotations

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Most Existing Methods handle occlusions poorly

- State-of-the-art: Region-growing (!) based methods
- EXACT Study[2] compares 15 methods; No clear winner
- Small airways are challenging
- Challenging to overcome occlusions

[2] Lo, P., et.al : Extraction of airways from CT (EXACT'09). IEEE Transactions on Medical Imaging, (2012)

Graph Refinement Model for Airway Extraction

Graph Refinement Model for Airway Extraction

Desired Properties

- Exploratory (to overcome occlusions)
- Detect small airways
- Uncertainty estimates

Preprocess Image to Graph Model

One possibility ....

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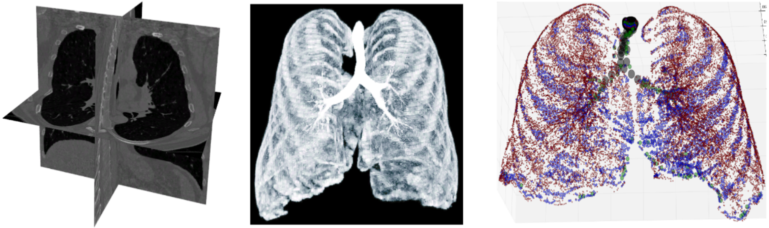

Figure 1: Visualisation of the pre-processing carried out to transform the input image (left) into a probability image (center) and then into graph format (right). Nodes in the graph are shown in scale to capture the variations in their local radius.

Graph Refinement Model

- Input graph: $\mathcal{G}_i : \{\mathcal{N}, \mathcal{E}_i\}$
- Node features: $\mathbf{X} \in \mathbb{R}^{F \times N}$
- Input adjacency: $\mathbf{A}_i \in \{0, 1\}^{N \times N}$

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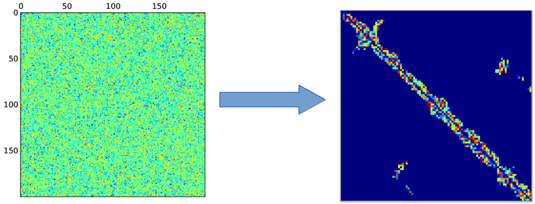
Graph Refinement Task

$$f(\mathcal{G}_i) \rightarrow \mathcal{G}$$

Output subgraph $\mathcal{G}$ with $\mathcal{E} \subset \mathcal{E}_i$; $\mathbf{A} \in \{0,1\}^{N \times N}$

Visualise Graph Refinement Task



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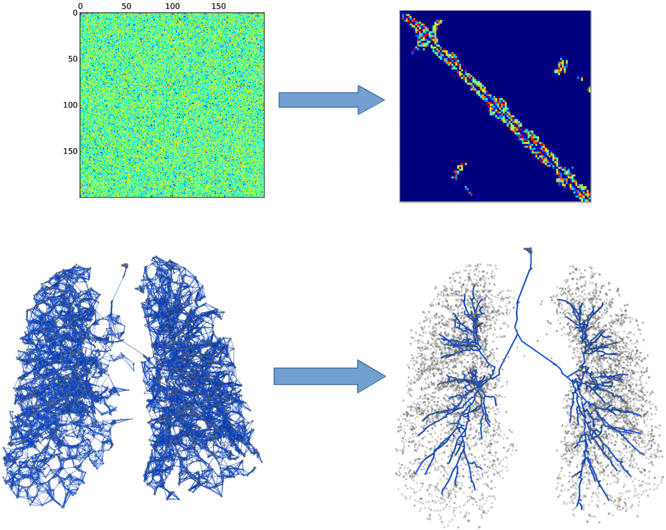

Updating Probabilistic Graphical Model

- Binary random variable to capture existence of edge between nodes
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$$\begin{aligned}\ln p(\mathbf{S}|\mathbf{X}) &\propto \ln p(\mathbf{S}, \mathbf{X}) \\ &= -\ln Z + \sum_{i \in \mathcal{N}} \phi_i(\mathbf{s}_i) + \sum_{(i,j) \in \mathcal{E}} \phi_{ij}(\mathbf{s}_i, \mathbf{s}_j),\end{aligned}$$

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Variational Approximate Inference

Minimize ELBO to obtain $q(\mathbf{S}) \in \mathcal{Q}$

$$\mathcal{F}(q_{\mathbf{S}}) = \ln Z + \mathbb{E}_{q_{\mathbf{S}}}\Big[\ln p(\mathbf{S}|\mathbf{X}) - \ln q(\mathbf{S})\Big]$$

Simpler approximation using MFA

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Factorisable $q(\mathbf{S})$

$$q(\mathbf{S}) = \prod_{i=1}^{N}\prod_{j=1}^{N} q_{ij}(s_{ij}),$$

$$\text{where, } q_{ij}(s_{ij}) = \begin{cases} \alpha_{ij} & \text{if } s_{ij} = 1 \\ (1-\alpha_{ij}) & \text{if } s_{ij} = 0 \end{cases},$$

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Assumes edges to be independent.

Node and Pairwise Potentials for MFA

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Node Potential

For each node $i \in \mathcal{N}$,

$$\phi_i(\mathbf{s}_i) = \sum_{v=0}^{D} \beta_v \mathbb{I}\Big[\sum_j s_{ij} = v\Big] + \mathbf{a}^T \mathbf{x}_i \sum_j s_{ij},$$

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Pairwise Potential

For each edge, $(i,j) \in \mathcal{E}_i$

$$\phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) = \lambda\big(1 - 2|s_{ij} - s_{ji}|\big) + (2s_{ij}s_{ji} - 1)\Big[\boldsymbol{\eta}^T|\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T(\mathbf{x}_i\mathbf{x}_j)\Big].$$

Parameters $= [\boldsymbol{\beta}, \mathbf{a}, \lambda, \boldsymbol{\eta}, \boldsymbol{\nu}]$

Minimize ELBO to get MFA Iterations

MFA Iterations

$$\alpha_{kl}^{(t+1)} = \sigma(\gamma_{kl}) = \frac{1}{1 + \exp^{-\gamma_{kl}}} \;\forall\; k = \{1 \dots N\},\; l \in \mathcal{N}_k,$$

where $\sigma(\cdot)$ is sigmoid activation, $\mathcal{N}_k$ are neighbours of node k, and

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$$\begin{aligned}\gamma_{kl} = & \prod_{j \in \mathcal{N}_k \setminus l} (1-\alpha_{kj}^{(t)}) \Bigg\{ \sum_{m \in \mathcal{N}_k \setminus l} \frac{\alpha_{km}^{(t)}}{(1-\alpha_{km}^{(t)})} \Big[(\beta_2 - \beta_1) \\ & - \beta_2 \sum_{n \in \mathcal{N}_k \setminus l,m} \frac{\alpha_{kn}^{(t)}}{(1-\alpha_{kn}^{(t)})} \Big] + (\beta_1 - \beta_0) \Bigg\} + \mathbf{a}^T \mathbf{x}_k \\ & + (4\alpha_{lk}^{(t)} - 2)\lambda + 2\alpha_{lk}^{(t)} (\boldsymbol{\eta}^T |\mathbf{x}_k - \mathbf{x}_l| + \boldsymbol{\nu}^T (\mathbf{x}_k \mathbf{x}_l)).\end{aligned} \tag{1}$$

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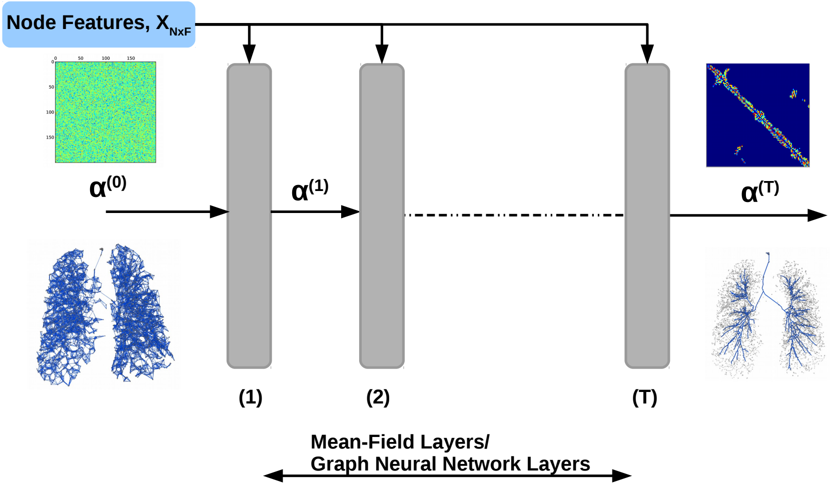
  $\boldsymbol{\alpha}$ is soft prediction of global connectivity variable
- $T-$iterations as a $T-$layered network
- Automatic differentiation to learn parameters: $\mathcal{L}(\boldsymbol{\alpha}, \mathbf{A}_r)$, where $\mathbf{A}_r$ is reference adjacency.

Summarising MFN



Summarising MFN

- Yields approximation to underlying posterior
- Simple factors
- Handful of parameters
- Easy to optimise
- – Hand-crafting potentials is cumbersome
- – Might not generalise across applications

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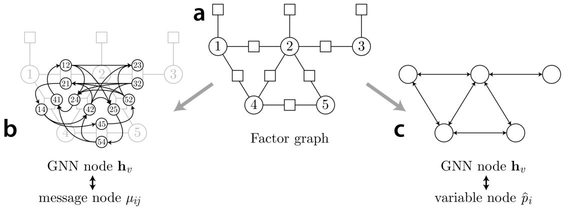

Figure: Two mappings of Factor Graphs into GNNs

Figure from Yoon et al. "Inference in probabilistic graphical models by Graph Neural Networks" (2018)

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GAE Model for Graph Refinement: Encoder

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Encoder:

$$
\begin{aligned}
\text{Node Embedding:} \quad & \mathbf{h}_j^1 = & g_n(\mathbf{x}_j) \\
\text{Node-to-Edge mapping:} \quad & \mathbf{h}_{(i,j)}^1 = & g_{n2e}([\mathbf{h}_i^1, \mathbf{h}_j^1]) \\
\text{Edge-to-Node mapping:} \quad & \mathbf{h}_j^2 = & g_{e2n}(\sum_i^{N_j} \mathbf{h}_{(i,j)}^1]) \\
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\end{aligned}
$$

$g_{\dots}(\cdot)$ are 2-layered MLPs, with ReLU, dropout and layer normalisation

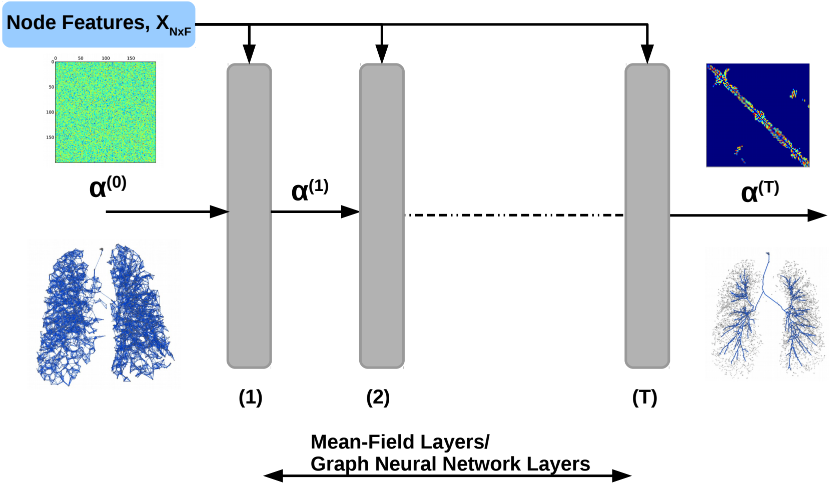
GAE Model for Graph Refinement: Decoder

Decoder:

$$\alpha_{ij} = \sigma(g_{dec}(\mathbf{h}^2_{(i,j)})) \tag{2}$$

g_{dec} is a 1D convolutional layer with one output channel
Model can be trained by computing the loss $\mathcal{L}\boldsymbol{\alpha}, \mathbf{A}_r$

Summarising GNNs



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Training both models: Dice Loss

To tackle Class Imbalance:

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$$\mathcal{L}(\boldsymbol{\alpha}, \mathbf{A}_r) = 1 - \frac{2\sum_{i,j=1}^{N} \alpha_{ij} A_{ij}}{\sum_{i,j=1}^{N} \alpha_{ij}^2 + \sum_{i,j=1}^{N} A_{ij}^2},$$

A_{ij} are individual binary entries $\mathbf{A}_r$

Data

- Danish Lung Cancer Screening Trial
- Low-dose Chest CT scans
- 32 scans with “Reference” annotations
- 100 scans with automatic segmentation

Results

- Error Measure based on centerline distances
- Average of two distances, $d_{err} = (d_{FP} + d_{FN})/2$
- Compared with Top Performer on Airway Extraction Challenge

Table I: Performance comparison

Method	d_{FP}(mm)	d_{FN}(mm)	d_{err} (mm)
Voxel Classifier	3.871	5.108	4.489 ± 0.754
MFN	3.716	3.992	3.845 ± 0.415
GNN	3.513	2.890	3.202 ± 0.386