CHALMERS

Bayesian tracking of multiple point targets

Using Expectation Maximization

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Introduction

Why the interest in target tracking?

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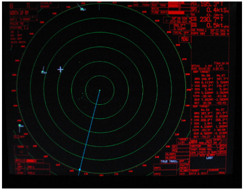

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  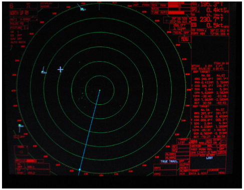

Challenging task, with a need for improved solutions.

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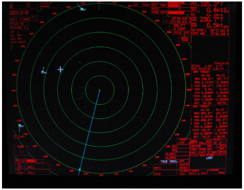

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Overview

Parameter Estimators

Parameter Estimators

If Y is the observed data and X are the parameters of interest, then

Maximum Likelihood (ML)

$$\hat{X}_{ML} = \arg\max_{X} p(Y|X) \equiv \ell(X|Y)$$

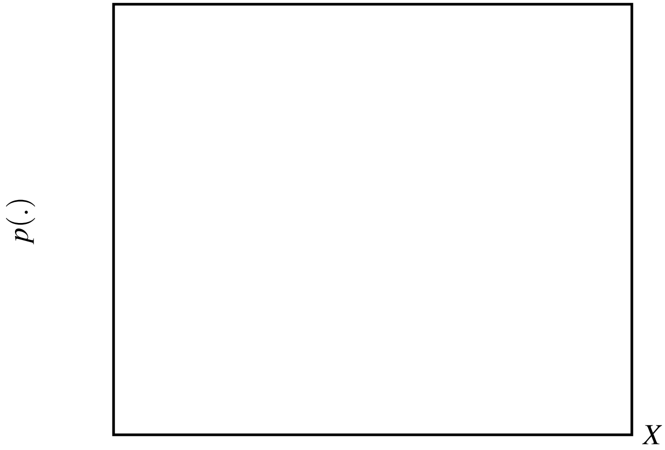
Bayesian estimation

$$p(X|Y) = \frac{p(X)p(Y|X)}{p(Y)}$$

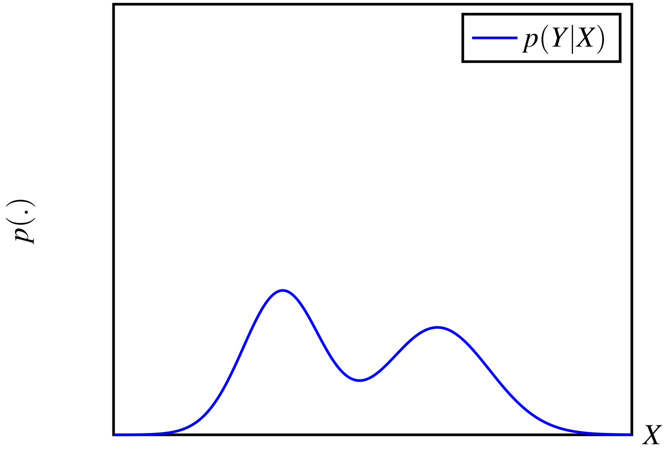
$$posterior \propto prior \times likelihood$$

Obtain point estimates from the posterior: Mean yields Minimum Mean Squared Error (MMSE) estimator, whereas, the mode yields Maximum A Posteriori estimator.

ML,MAP and MMSE estimators



ML,MAP and MMSE estimators



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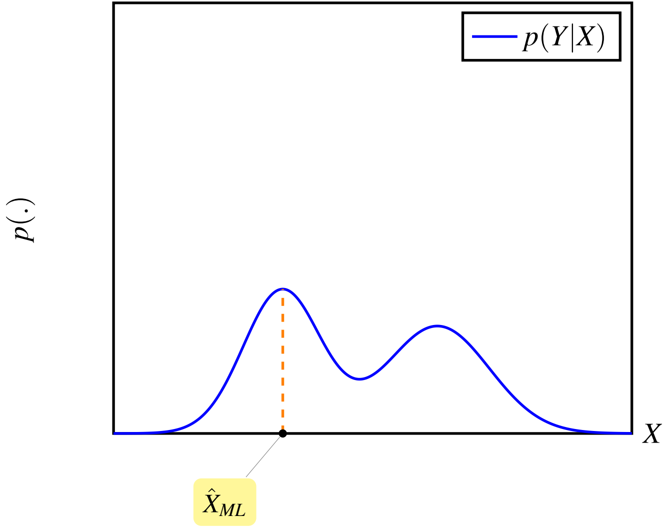

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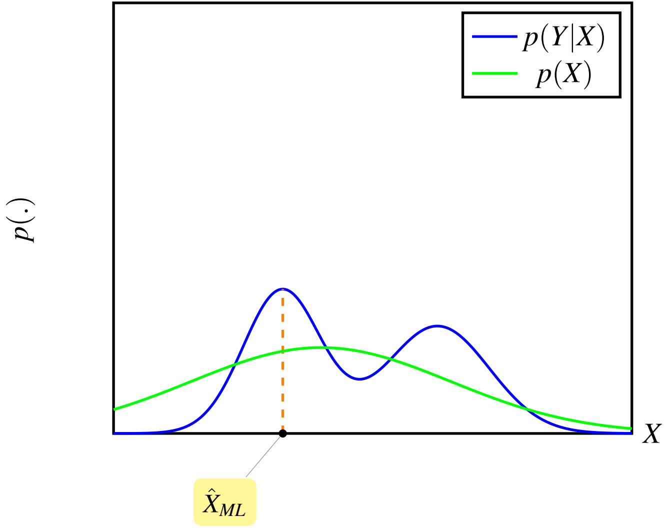

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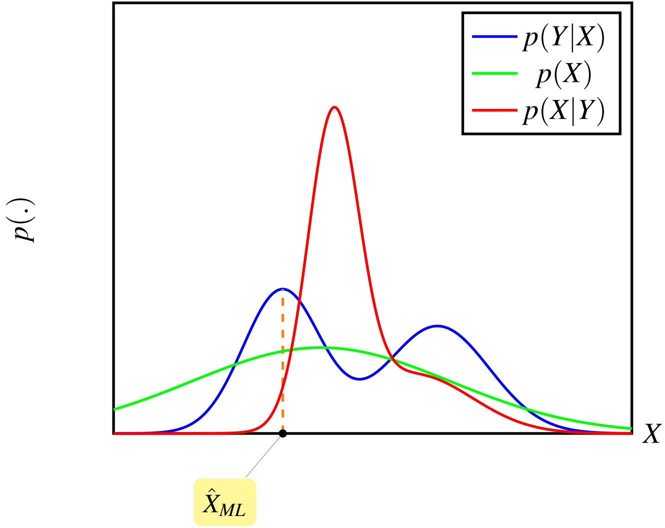

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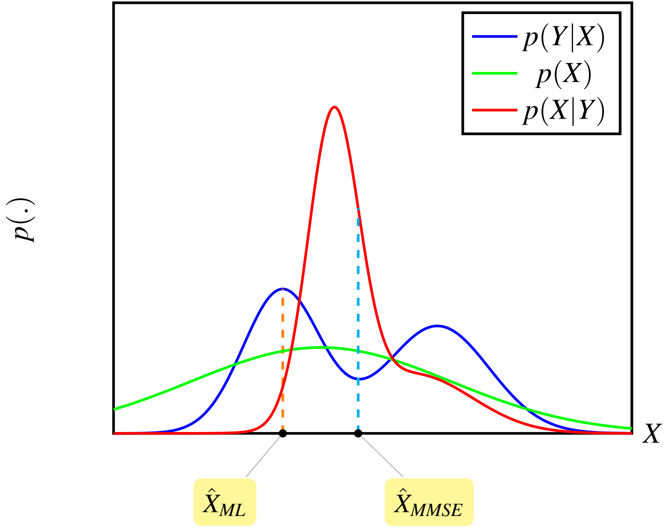

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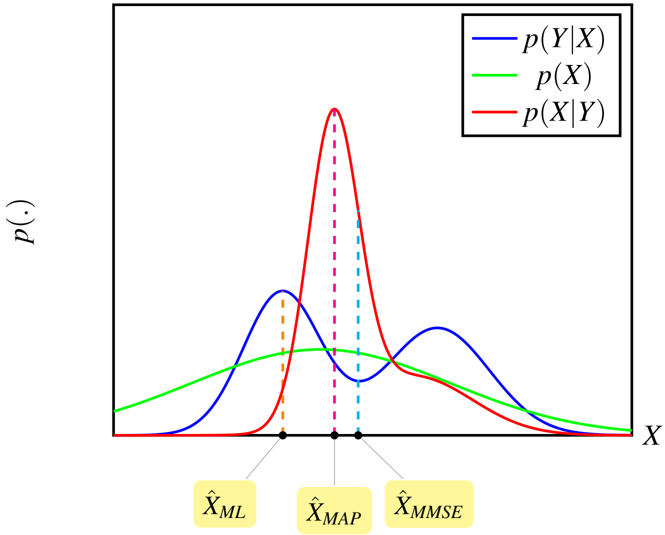

Parameter estimation-II

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Sequential parameter estimation

Estimating parameters from sequentially obtained data, generated by a dynamical systems, which change state with time.

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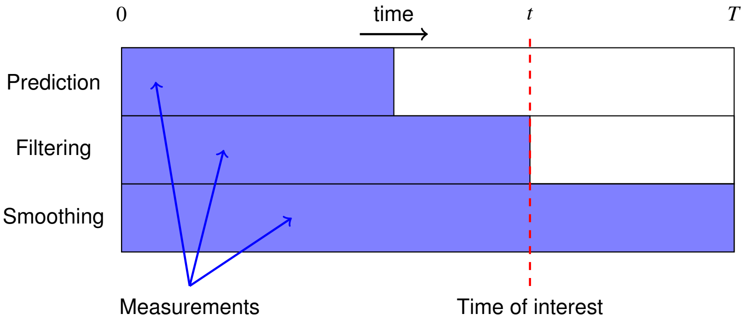
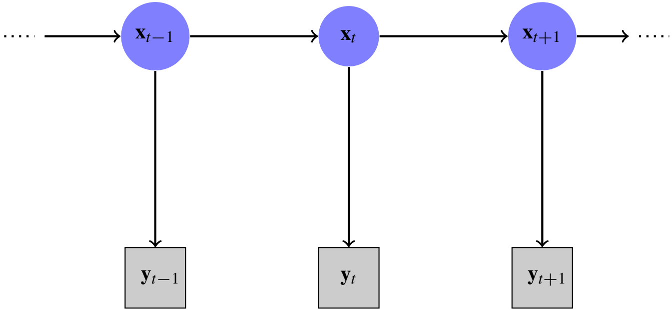


Uncertainties in measurement origin



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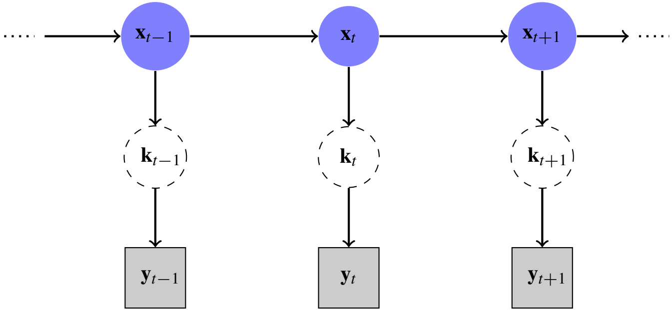

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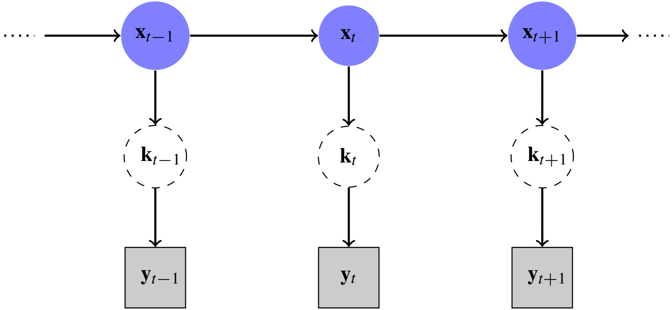


- When performing sequential parameter estimation, uncertainties in measurement origin, if any, must be first resolved in order to estimate the parameters.
- Data association uncertainties can be interpreted as hidden and unobserved variables.

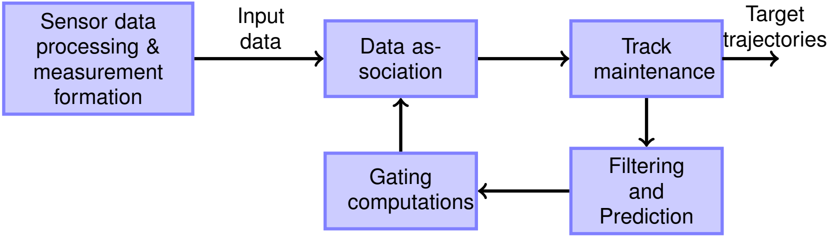
Data association uncertainties

There are many reasons for uncertainties in data association.

- Measurement noise, R.
- Clutter measurements, β_c.
- Missed detections, P_D.
- Sensor resolution.

One of the primary challenges in multi-target tracking, hence also the main focus in this thesis.

Multi-Target Tracking



Optimal data association solution

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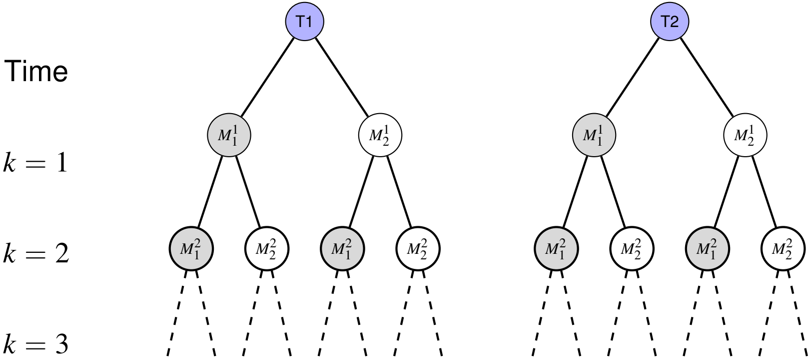


Optimal data association solution

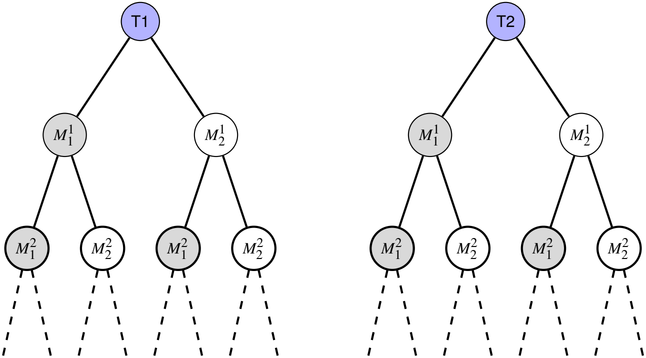
Maintaining hypothesis tree is a common approach.
Hypothesis tree for a two target case.

Time

$k = 1$

$k = 2$

$k = 3$



Optimal solution is intractable!

Problem and solutions

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Problem formulation

To estimate target trajectories from the obtained measurements, after resolving the problem of data association uncertainties, while satisfying point target model assumptions.

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Existing MTT solutions approximate the optimal solution.

- Pruning data association hypotheses – Global Nearest Neighbour (GNN).
- Merging them – Joint Probabilistic Data Association (JPDA).
- Deferred decision plus pruning – Multiple Hypothesis Tracking (MHT).
- Iterative optimization – Probabilistic MHT (PMHT).

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We perform iterative optimization, using pruning and merging.

Expectation Maximization (EM)

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The EM algorithm

Iterative algorithm used to approximate MAP/ML estimates from incomplete data, or models with hidden variables.

- The idea is natural and intuitive.
- Particularly useful when dealing with models with *hidden variables*.
- Lower bounds the objective function being evaluated.
- Guarantees convergence (at least to a local optimum).
- MTT solutions using PMHT already exist.

Lower bounding the objective function

Lower bounding the objective function

MAP estimation from a model with hidden variables K,

$$\hat{X} = \arg\max_X p(X|Y) \equiv \arg\max_X \ln p(X,Y)$$

Objective function $\ln p(X|Y)$ is not tractable, due to the presence of hidden variables.

$$\begin{aligned}\ln p(X,Y) &= \ln \sum_K p(X,K,Y) \\ &= \ln \sum_K q_K(K) \frac{p(X,K,Y)}{q_K(K)} \\ &\geq \sum_K q_K(K) \ln \frac{p(X,K,Y)}{q_K(K)} \triangleq \mathcal{F}(q_K, X)\end{aligned}$$

Using Jensen's inequality: For a concave function, $f(\mathbf{E}(x)) \geq \mathbf{E}(f(x))$.

E and M steps

The lower bound has two free variables $\mathcal{F}(q_K, X)$. E and M steps correspond to the *iterative* maximization of the lower bound wrt $q_K(K)$ and X, while keeping the other fixed.

E-step

$$
\begin{aligned}
q_K^{(n+1)}(K) &= \arg\max_{q_K} \mathcal{F}(q_K, X^{(n)}) \\
&= \Pr\{K|X^{(n)}, Y\}
\end{aligned}
$$

M-step

$$
\begin{aligned}
\hat{X}^{(n+1)} &= \arg\max_{X} \mathcal{F}(q_K^{(n+1)}, X) \\
&= \arg\max_{X} \sum_{K} \Pr\{K|X^{(n)}, Y\} \ln p(X, K, Y).
\end{aligned}
$$

EM in action

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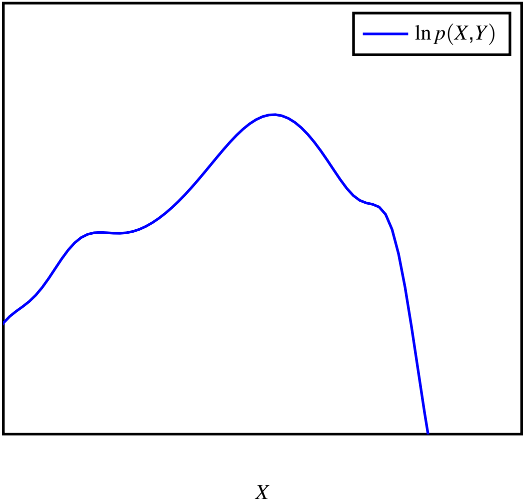

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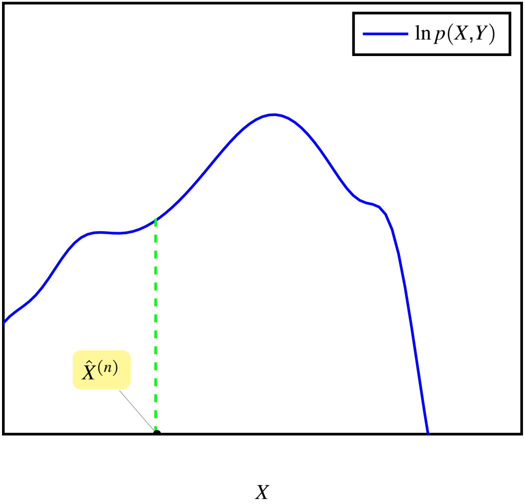

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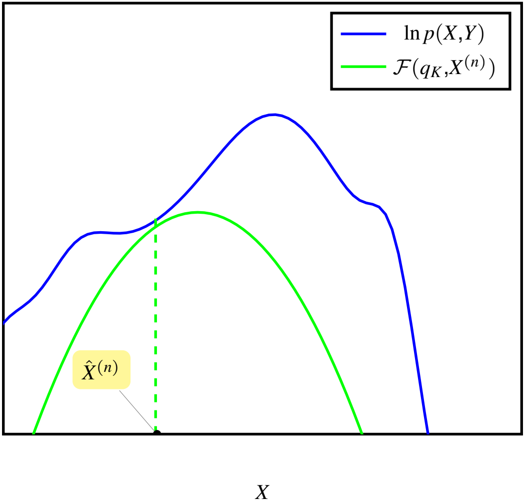

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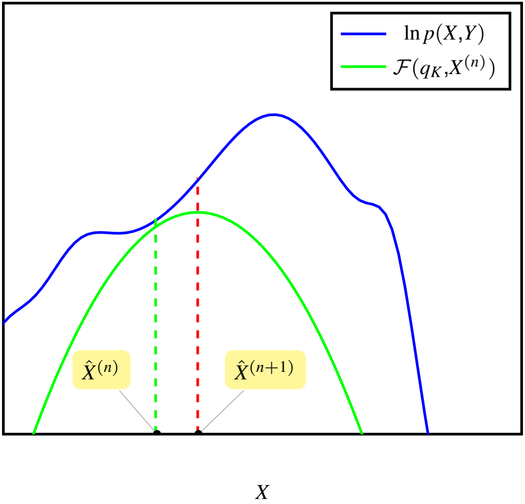

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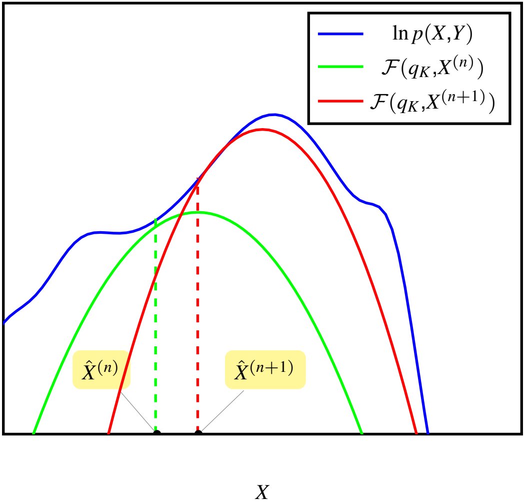

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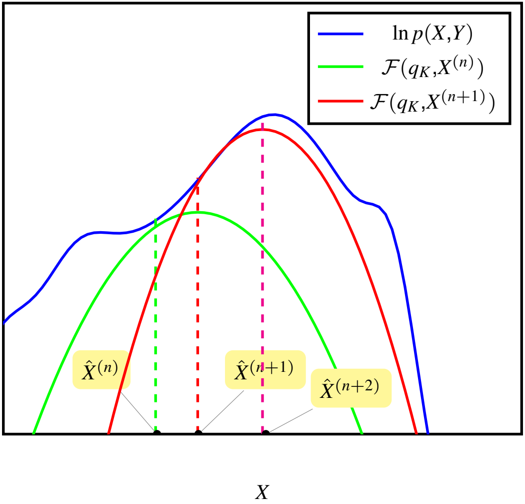

Contributions

Two algorithms, for tracking multiple point targets, based on EM. Using EM terminology,

Proposed Algorithm I

Data association uncertainties are treated as hidden variables, and target states as parameters of interest. Thus, density over data association variables and point estimates of target states.

Proposed Algorithm II

Target states are treated as hidden variables, and data association variables as parameters of interest. Thus, density over target states and point estimates of data association.

The resulting algorithms can be implemented using simpler, standard solution blocks like smoothing, auction algorithm and by computing marginal probabilities.

Proposed Algorithm I

Overview of the algorithm:

- Data association variable is treated as hidden.
- Point estimates of target states.
- Close to PMHT, as both use EM.
- Different from PMHT, as PMHT assumes extended targets.
- Implemented using smoothing and computation of marginal data association probabilities.

E & M steps in Proposed Algorithm I

E and M steps

$$q_K^{(n+1)}(K) = \Pr\{K|X^{(n)}, Y\}$$
$$\hat{X}^{(n+1)} = \arg\max_X \sum_K \Pr\{K|X^{(n)}, Y\} \ln p(X, K, Y).$$

- Computing the data association probabilities, for all possible hypotheses is expensive. Instead, approximate marginal data association probabilities are computed.
- M-step using RTS smoother.

Marginal data association probabilities

Hyp.	T1	T2	T3	Hyp. Prob.
H_1	1	2	3	0.35
H_2	1	3	2	0.1
H_3	2	1	3	0.25
H_4	2	3	1	0.05
H_5	3	1	2	0.15
H_6	3	2	1	0.2

$w_{11} = 0.35 + 0.1 = 0.45$

$w_{12} = 0.25 + 0.05 = 0.3$

$w_{13} = 0.15 + 0.2 = 0.35$

Table: Illustration of marginal probabilities calculation

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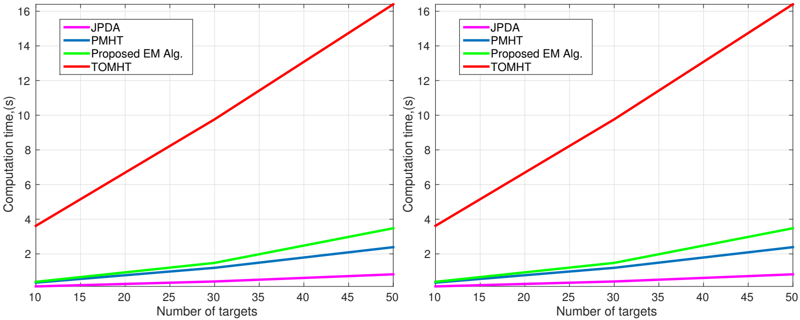
This brute force approach is intractable for large number of targets. In the proposed algorithm, marginal probabilities are approximated using Loopy Belief Propagation.

Results


JPDA
PMHT
Proposed EM Alg.
TOMHT
Computation time,(s)
Number of targets
16
14
12
10
8
6
4
2
10
15
20
25
30
35
40
45
50

Proposed algorithm II

Overview of the algorithm:

- Target states are treated as hidden.
- Point estimation of data association variable.
- Can be interpreted as iterative GNN, performing local optimizations.
- Implemented using smoothing and 2-D auction algorithm.

E & M steps in Proposed Algorithm II

E and M steps

$$q_K^{(n+1)}(X) = p(X|K^{(n)}, Y)$$
$$\hat{K}^{(n+1)} = \arg\max_K \int p(X|K^{(n)}, Y) \ln p(X, K, Y) dX.$$

- No need to compute marginal data association probabilities.
- E-step easily implemented using RTS smoother.
- M-step can be implemented using 2-D auction algorithm.

2-D auction algorithm

	Measurement 1	Measurement 2	Measurement 3
Target 1	-1	-10	-8
Target 2	-4	-12	-7
Target 3	-10	-5	-15

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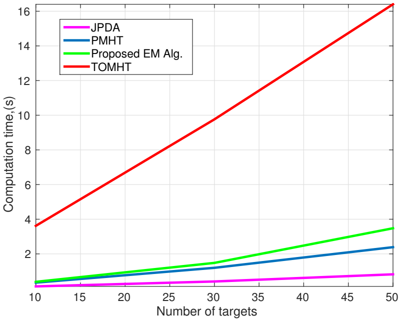
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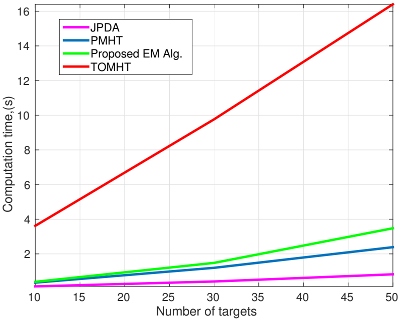
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- Significant computational advantage over TOMHT.

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- Relating the solutions to the Variational Bayesian framework.

Thank you for listening.