Extraction of Airways from Volumetric Data

PhD Thesis Defence

Raghavendra Selvan
raghav@di.ku.dk

Supervisors: Marleen de Bruijne, Jens Petersen

Department of Computer Science

Guess Who

https://www.copdfoundation.org/

Guess Who

Leonard Nimoy aka Spock
(1931-2015)

https://www.copdfoundation.org/

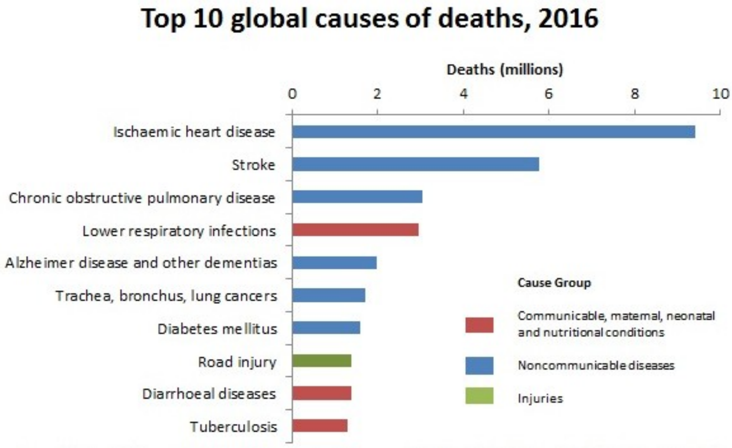
Guess Who

Leonard Nimoy aka Spock
(1931-2015)

Chronic Obstructive Pulmonary Disease (COPD)

https://www.copdfoundation.org/

Respiratory diseases: Major cause of morbidity & mortality



Source: Global Health Estimates 2016, World Health Organization, 2018

Outline

Outline

1. Airway diseases and diagnosis
2. Objective of the study
3. Data
4. Contributions
   - Multiple Hypothesis Tracking
   - Bayesian Smoothing
   - Graph Refinement Models
5. Summary & Conclusions

Respiratory diseases adversely affect airways

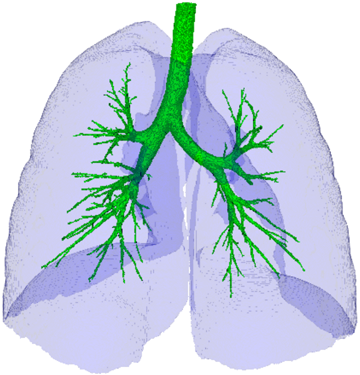


Image adapted from Wikimedia Commons

Respiratory diseases adversely affect airways

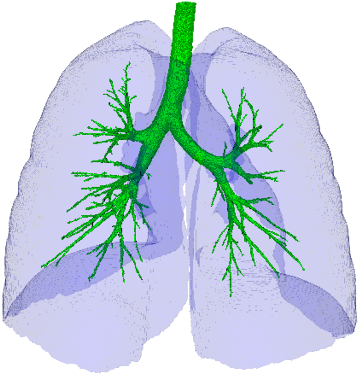
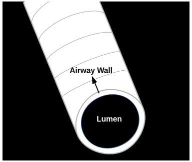


Image adapted from Wikimedia Commons

Particularly, airway morphology



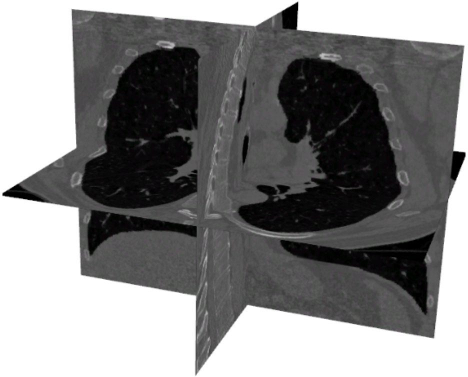
Existing diagnostics are rudimentary

Image sourced from Wikimedia Commons

Lung Function Tests

+ Non-invasive
+ Inexpensive
+ Reliable, mostly
– Little or no insight on pathophysiology
– Patient dependent
– Low reproducibility
– Mild cases can go unnoticed

Imaging based Computer Aided Diagnosis



Computed Tomography (CT) chest scans

- High-resolution imaging
- Pathophysiology can be studied
- Possibility of automated analysis

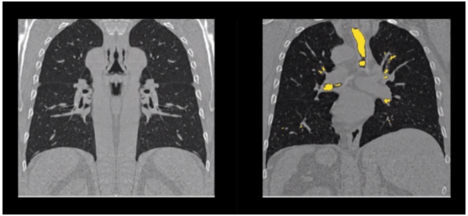
Outline

1. Airway diseases and diagnosis
2. Objective of the study
3. Data
4. Contributions
   - Multiple Hypothesis Tracking
   - Bayesian Smoothing
   - Graph Refinement Models
5. Summary & Conclusions

Imaging based analysis of airways & challenges

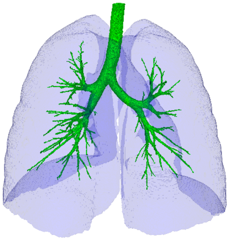
Three primary steps:

1. **Detection** of airways
2. Measurement of airway morphology
3. Deriving biomarkers



Coronal view of chest CT scan

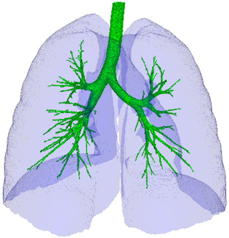
Methods exist. Majority of them are sequential



Sequential segmentation methods

Lo, P., et.al : Extraction of airways from CT (EXACT'09). IEEE Transactions on Medical Imaging, (2012)

Methods exist. Majority of them are sequential

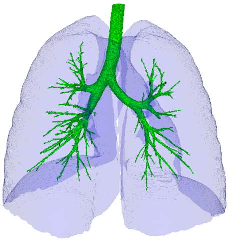


Sequential segmentation methods

- Susceptible to occlusions in data
- Small branches are challenging
- EXACT'09 Study
  - Airway extraction challenge
  - Compares 15 methods
  - 10 use region growing!

Lo, P., et.al : Extraction of airways from CT (EXACT'09). IEEE Transactions on Medical Imaging, (2012)

Methods exist. Majority of them are sequential



Sequential segmentation methods

- Susceptible to occlusions in data
- Small branches are challenging
- EXACT'09 Study
  - Airway extraction challenge
  - Compares 15 methods
  - 10 use region growing!

Lo, P., et.al : Extraction of airways from CT (EXACT'09). IEEE Transactions on Medical Imaging, (2012)

Objective of this thesis

Extraction of airways from volumetric data

With automatic methods that:

- Are exploratory
- Use more global information in local decisions

Outline

1. Airway diseases and diagnosis
2. Objective of the study
3. Data
4. Contributions
   - Multiple Hypothesis Tracking
   - Bayesian Smoothing
   - Graph Refinement Models
5. Summary & Conclusions

Data from Danish Lung Cancer Screening Trial (DLCST)

- $> 10,000$ Low-dose CT from 2052 subjects
- Smoker or former smoker (> 20 pack years)
- Voxels $\sim 0.75 \times 0.75 \times 1$ mm^3

Pedersen, J. H., et.al : The Danish randomized lung cancer CT screening trial – Overall design and results of the prevalence round. Journal of Thoracic Oncology, (2009)

Outline

1. Airway diseases and diagnosis
2. Objective of the study
3. Data
4. Contributions
   - Multiple Hypothesis Tracking
   - Bayesian Smoothing
   - Graph Refinement Models
5. Summary & Conclusions

Multiple Hypothesis Tracking (MHT)

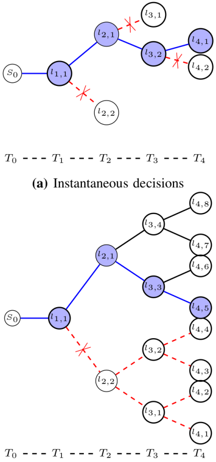
Work based on

[1] Raghavendra Selvan, Jens Petersen, Jesper H. Pedersen, and Marleen de Bruijne. "Extracting Tree-structures in CT data by Tracking Multiple Statistically Ranked Hypotheses" (2018). (Under review)

[2] Raghavendra Selvan, Jens Petersen, Jesper H. Pedersen, and Marleen de Bruijne. "Extraction of airway trees using multiple hypothesis tracking and template matching". In The Sixth International Workshop on Pulmonary Image Analysis. MICCAI, 2016.

Improvements to an established method

- MHT is extensively used in object tracking
- *Interactive* vessel segmentation method (Friman et al. 2010)
- Modifications render it *automatic*; suitable for airway tree extraction
  - New scale-invariant statistic
  - Improved bifurcation handling
- Significant performance improvement



(a) Instantaneous decisions

(b) Deferred decisions

Friman, Ola, et al. "Multiple hypothesis template tracking of small 3D vessel structures." Medical image analysis 14.2 (2010): 160-171.

Bayesian Smoothing

Work based on

[1] Raghavendra Selvan, Jens Petersen, Jesper H. Pedersen, and Marleen de Bruijne. "Extraction of airways with probabilistic state-space models and Bayesian smoothing." In Graphs in Biomedical Image Analysis, Computational Anatomy and Imaging Genetics, MICCAI, 2017, pp. 53-63. Springer, Cham.

Airway extraction with Bayesian smoothing

Idea

- Track candidate branches from across the volume
- Use uncertainty measures to qualify branches

Airway extraction with Bayesian smoothing

Idea

- Track candidate branches from across the volume
- Use uncertainty measures to qualify branches

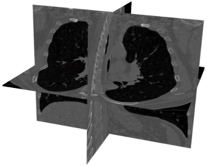
State-space models on sparse point cloud data

Airway extraction with Bayesian smoothing

Idea

- Track candidate branches from across the volume
- Use uncertainty measures to qualify branches

State-space models on sparse point cloud data



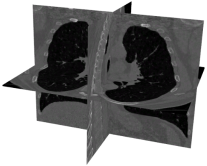
Dense Volume

Airway extraction with Bayesian smoothing

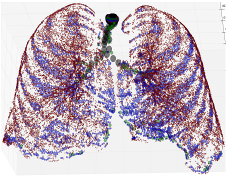
Idea

- Track candidate branches from across the volume
- Use uncertainty measures to qualify branches

State-space models on sparse point cloud data



Dense Volume



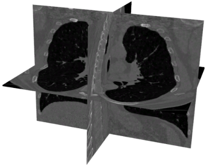
Sparse point cloud

Airway extraction with Bayesian smoothing

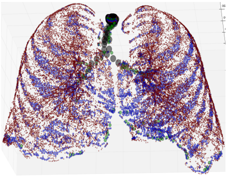
Idea

- Track candidate branches from across the volume
- Use uncertainty measures to qualify branches

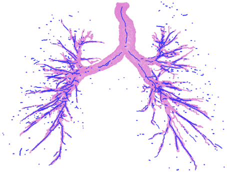
State-space models on sparse point cloud data



Dense Volume



Sparse point cloud



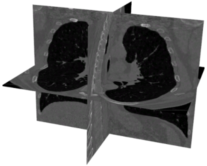
Tracked branches

Airway extraction with Bayesian smoothing

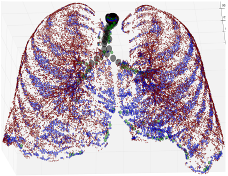
Idea

- Track candidate branches from across the volume
- Use uncertainty measures to qualify branches

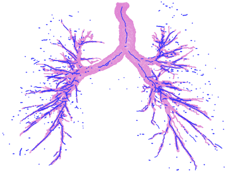
State-space models on sparse point cloud data



Dense Volume



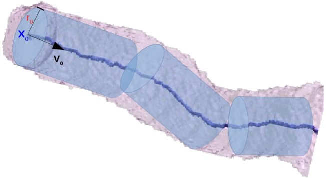
Sparse point cloud



Tracked branches

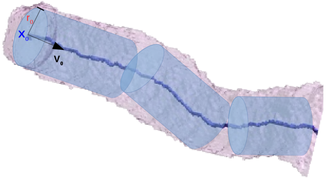
Qualified branches

State-space model



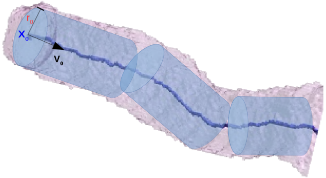
- Airway tree as a set of *independent* branches
  $\mathbf{X} = \{\mathbf{X}_1, \dots, \mathbf{X}_T\}$

State-space model



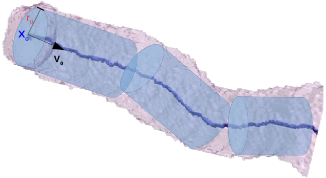
- Airway tree as a set of *independent* branches
  $\mathbf{X} = \{\mathbf{X}_1, \ldots, \mathbf{X}_T\}$
- Each branch as a sequence of state vectors
  $\mathbf{X}_i = [\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{l_i}]$

State-space model

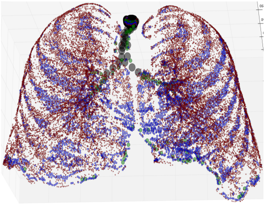


- Airway tree as a set of *independent* branches $\mathbf{X} = \{\mathbf{X}_1, \dots, \mathbf{X}_T\}$
- Each branch as a sequence of state vectors $\mathbf{X}_i = [\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{l_i}]$
- State vector at each step $\mathbf{x}_k = [x, y, z, r, v_x, v_y, v_z]^T$

State-space model



- Airway tree as a set of *independent* branches $\mathbf{X} = \{\mathbf{X}_1, \ldots, \mathbf{X}_T\}$
- Each branch as a sequence of state vectors $\mathbf{X}_i = [\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{l_i}]$
- State vector at each step $\mathbf{x}_k = [x, y, z, r, v_x, v_y, v_z]^T$



- Sparse, vectorised image data $\mathbf{Y} = [\mathbf{y}_0, \ldots, \mathbf{y}_T]$; $\mathbf{y}_k = [x, y, z, r]^T$

Probabilistic state-space models

Probabilistic state-space models

Process model

$$p(\mathbf{x}_k|\mathbf{x}_{k-1}) \equiv \mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{q} \tag{1}$$

$\mathbf{F}$: State transition function, $\mathbf{q} \sim N(\mathbf{0}, \mathbf{Q})$: Process noise

Probabilistic state-space models

Process model

$$p(\mathbf{x}_k|\mathbf{x}_{k-1}) \equiv \mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{q} \quad (1)$$

$\mathbf{F}$: State transition function, $\mathbf{q} \sim N(\mathbf{0}, \mathbf{Q})$: Process noise

Measurement model

$$p(\mathbf{y}_k|\mathbf{x}_k) \equiv \mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{m} \quad (2)$$

$\mathbf{H}$: Measurement function, $\mathbf{m} \sim N(\mathbf{0}, \mathbf{R})$: Measurement noise

Extraction of branches from posterior distribution

[5] Rauch-Tung-Striebel

Extraction of branches from posterior distribution

Estimation of $p(\mathbf{X}|\mathbf{Y})$

$$p(\mathbf{X}|\mathbf{Y}) \approx \prod_{i}^{T} p(\mathbf{X}_i|\mathbf{Y}) \tag{3}$$

[5] Rauch-Tung-Striebel

Extraction of branches from posterior distribution

Estimation of $p(\mathbf{X}|\mathbf{Y})$

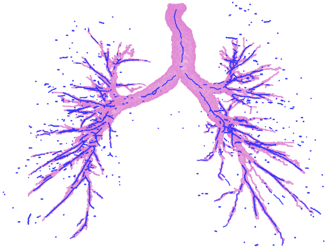
$$p(\mathbf{X}|\mathbf{Y}) \approx \prod_{i}^{T} p(\mathbf{X}_i|\mathbf{Y}) \tag{3}$$

Recursive estimation of $p(\mathbf{X}_i|\mathbf{Y})$ using RTS[5] smoother

- Off-the-shelf Bayesian smoother
- Closed form, simple-to-compute
- Gaussian density estimates at each step
- Inherent uncertainty measure

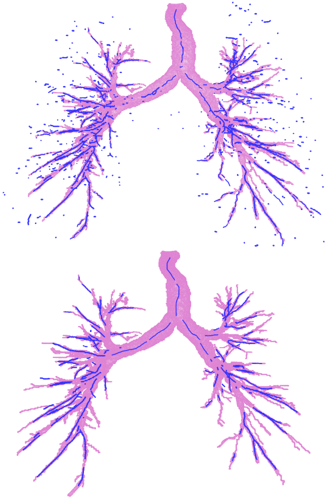
[5] Rauch-Tung-Striebel

Qualification of tracked branches



- Exploratory nature → Several candidate branches
- Qualify branches based on posterior covariance
- Measures branch fitness to the model

Qualification of tracked branches



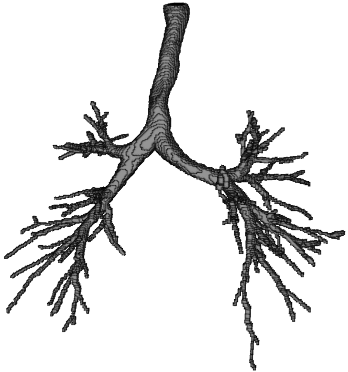
- Exploratory nature $\rightarrow$ Several candidate branches
- Qualify branches based on posterior covariance
- Measures branch fitness to the model

$$\mu_i = \frac{\sum_{k=1}^{l_i} \mathrm{Tr}(\mathbf{P}_{k|k})}{l_i}. \tag{4}$$

$\mathbf{P}_{k|k}$ is posterior covariance matrix at step k.

Data

- Data from DLCST
- Reference dataset (32 scans)
- Additional 100 scans; automatic segmentations



Pedersen, J. H., et.al : The Danish randomized lung cancer CT screening trialoverall design and results of the prevalence round. Journal of Thoracic Oncology, (2009)

Preprocessing of data

- Trained voxel classifier to obtain probability images (Lo et al.2010)
- Multi-scale Laplacian of Gaussians to obtain sparse point cloud

Lo, Pechin, et al. "Vessel-guided airway tree segmentation: A voxel classification approach." Medical image analysis 14.4 (2010): 527-538.

Experiments

- **Baseline:** Region growing on probability images
- Bayesian smoothing merged with region growing for evaluation
- Eight-fold cross validation

Experiments

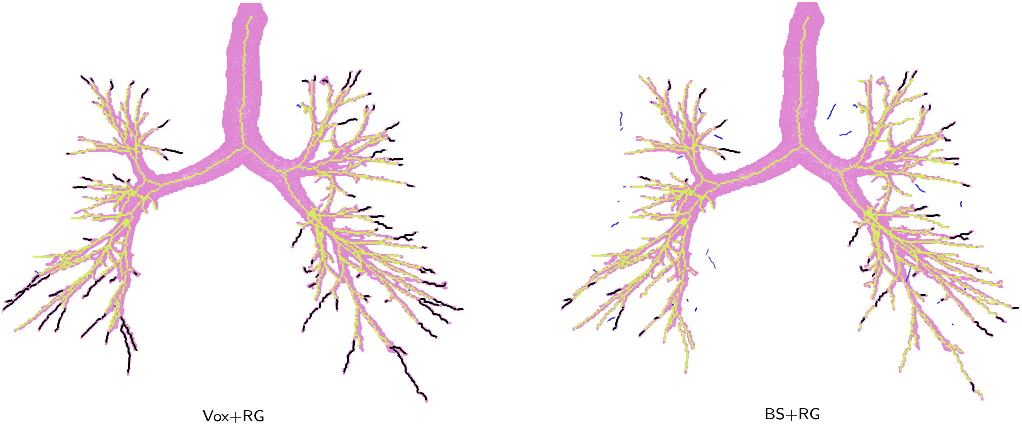
- **Baseline:** Region growing on probability images
- Bayesian smoothing merged with region growing for evaluation
- Eight-fold cross validation
- **Error measures**:
    - Average centerline distance: $d_{err} = (d_{FP} + d_{FN})/2$
    - $d_{FP} \equiv$ Specificity
    - $d_{FN} \equiv$ Sensitivity
    - Percentage of tree length (TL)
    - False positive rate (FPR)

Performance comparison

	d_{FP}(mm)	d_{FN}(mm)	d_{err} (mm)	TL.(%)	FPR(%)
Vox+RG	3.624 ± 0.776	5.155 ± 0.580	4.389 ± 0.441	79.6 ± 7.2	5.0 ± 3.9
BS+RG	3.921 ± 0.612	4.218 ± 0.334	4.069 ± 0.476	82.3 ± 6.1	8.7 ± 3.4

- $d_{FP} \equiv$ Specificity
- $d_{FN} \equiv$ Sensitivity
- Average centerline distance: d_{err}
- Percentage of tree length (TL)
- False positive rate (FPR)

Visualisation of extracted airways



Legend: Reference (pink), True Positive (Yellow), False Negative (Black), False Positive (Blue)

Summary

+ Airway extraction in probabilistic state-space model setting
+ Bayesian smoothing method to track branches
+ Exploratory algorithm
+ Uncertainty estimates used to validate branches
+ Multivariate Gaussian density estimates /node/branch
– Increase in false positives
– Disconnected branches

Graph Refinement Models

Work based on

[1] Raghavendra Selvan, Thomas Kipf, Max Welling, Jesper H. Pedersen, Jens Petersen, and Marleen de Bruijne. "Graph Refinement based Tree Extraction using Mean-Field Networks and Graph Neural Networks" (2018). (In progress)

[2] Raghavendra Selvan, Max Welling, Jesper H. Pedersen, Jens Petersen, and Marleen de Bruijne. "Mean field network based graph refinement with application to airway tree extraction." 21st Conference on Medical Image Computing & Computer Assisted Intervention (MICCAI 2018), pp. 750-758, Cham. Springer International Publishing.

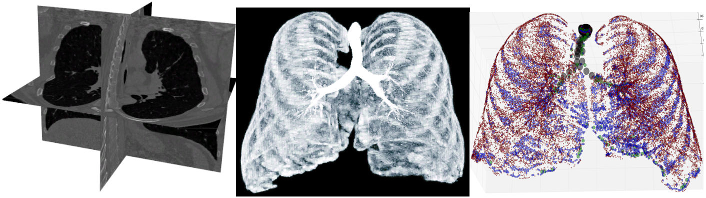
[3] Raghavendra Selvan, Thomas Kipf, Max Welling, Jesper H. Pedersen, Jens Petersen, and Marleen de Bruijne. "Extraction of Airways using Graph Neural Networks." 1st Conference on Medical Imaging with Deep Learning (MIDL 2018), Amsterdam.

Graph Refinement Model for Airway Extraction

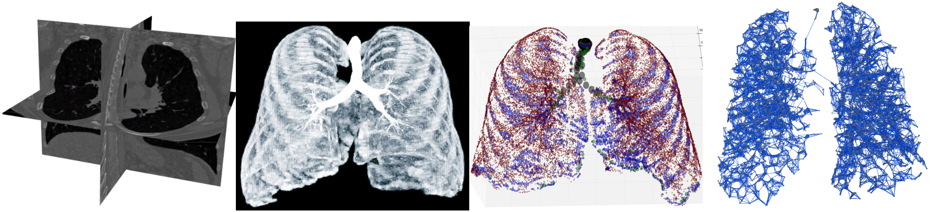
Motivation

- Building on Bayesian smoothing method
- Graphs with features derived from Gaussian density
- Optimise global connectivity, instead of qualifying individual branches

Volumetric data to Graph data



Volumetric data to Graph data



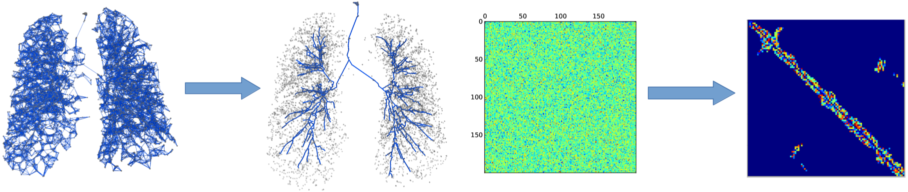
- Overconnected input graph: $\mathcal{G}_{\text{in}} : \{\mathcal{V}, \mathcal{E}_{\text{in}}\}$, with $|\mathcal{V}| = N$
- Node features: $\mathbf{X} \in \mathbb{R}^{F \times N}$
- Input adjacency: $\mathbf{A}_{\text{in}} \in \{0, 1\}^{N \times N}$

Airway extraction as Graph Refinement task

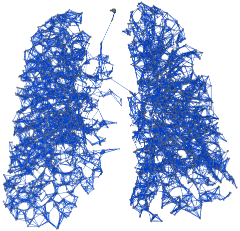
Graph Refinement Model

$$f : \mathcal{G}_{\text{in}} \rightarrow \mathcal{G}$$

Output subgraph $\mathcal{G}$ with $\mathcal{E} \subset \mathcal{E}_{in}$; $\mathbf{A} \in \{0, 1\}^{N \times N}$

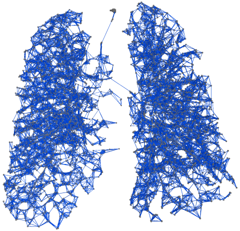


Probabilistic Graphical Model for MFN



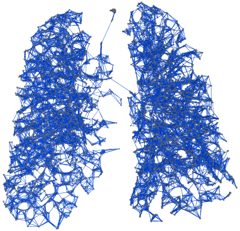
- Binary random variable
  $s_{ij} \in \{0, 1\}$ with prob. $\alpha_{ij} \in [0, 1]$

Probabilistic Graphical Model for MFN



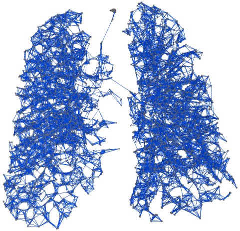
- Binary random variable $s_{ij} \in \{0,1\}$ with prob. $\alpha_{ij} \in [0,1]$
- For each node: $\mathbf{s}_i = \{s_{ij}\} : j = 1 \dots N$

Probabilistic Graphical Model for MFN



- Binary random variable $s_{ij} \in \{0, 1\}$ with prob. $\alpha_{ij} \in [0, 1]$
- For each node: $\mathbf{s}_i = \{s_{ij}\} : j = 1 \ldots N$
- Global connectivity variable: $\mathbf{S} = [\mathbf{s}_1 \ldots \mathbf{s}_N]$
- Instances of $\mathbf{S}$ are $N \times N$ adjacency matrices

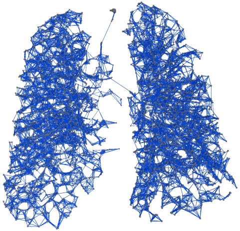
Probabilistic Graphical Model for MFN



- Binary random variable $s_{ij} \in \{0,1\}$ with prob. $\alpha_{ij} \in [0,1]$
- For each node: $\mathbf{s}_i = \{s_{ij}\} : j = 1 \dots N$
- Global connectivity variable: $\mathbf{S} = [\mathbf{s}_1 \dots \mathbf{s}_N]$
- Instances of $\mathbf{S}$ are $N \times N$ adjacency matrices

Posterior density of interest: $p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\text{in}})$

Probabilistic Graphical Model for MFN



- Binary random variable $s_{ij} \in \{0,1\}$ with prob. $\alpha_{ij} \in [0,1]$
- For each node: $\mathbf{s}_i = \{s_{ij}\} : j = 1 \ldots N$
- Global connectivity variable: $\mathbf{S} = [\mathbf{s}_1 \ldots \mathbf{s}_N]$
- Instances of $\mathbf{S}$ are $N \times N$ adjacency matrices

Posterior density of interest: $p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\text{in}})$

$$\begin{aligned}\ln p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\text{in}}) &\propto \ln p(\mathbf{S}, \mathbf{X}, \mathbf{A}_{\text{in}}) \\ &= \sum_{i \in \mathcal{N}} \phi_i(\mathbf{s}_i) + \sum_{(i,j) \in \mathcal{E}} \phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) - \ln Z,\end{aligned}$$

Node and Pairwise Potentials for MFA

Node Potential: For each node $i \in \mathcal{V}$

$$\phi_i(\mathbf{s}_i) = \sum_{v=0}^{D} \beta_v \mathbb{I}\left[\sum_j s_{ij} = v\right] + \mathbf{a}^T \mathbf{x}_i \sum_j s_{ij}, \tag{5}$$

Pairwise Potential: For each edge, $(i,j) \in \mathcal{E}_{\text{in}}$

$$\phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) = \lambda(1 - 2|s_{ij} - s_{ji}|) + (2 s_{ij} s_{ji} - 1)\left[\boldsymbol{\eta}^T |\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T (\mathbf{x}_i \mathbf{x}_j)\right]. \tag{6}$$

$$\text{Parameters} = [\cdot]$$

Node and Pairwise Potentials for MFA

Node Potential: For each node $i \in \mathcal{V}$

$$\phi_i(\mathbf{s}_i) = \sum_{v=0}^{D} \beta_v \mathbb{I}\left[\sum_j s_{ij} = v\right] + \mathbf{a}^T \mathbf{x}_i \sum_j s_{ij}, \quad (5)$$

Pairwise Potential: For each edge, $(i,j) \in \mathcal{E}_{\text{in}}$

$$\phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) = \lambda(1 - 2|s_{ij} - s_{ji}|) + (2s_{ij}s_{ji} - 1)\left[\boldsymbol{\eta}^T|\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T(\mathbf{x}_i\mathbf{x}_j)\right]. \quad (6)$$

Parameters $= [\beta]$

Node and Pairwise Potentials for MFA

Node Potential: For each node $i \in \mathcal{V}$

$$\phi_i(\mathbf{s}_i) = \sum_{v=0}^{D} \beta_v \mathbb{I}\left[\sum_j s_{ij} = v\right] + \mathbf{a}^T \mathbf{x}_i \sum_j s_{ij}, \tag{5}$$

Pairwise Potential: For each edge,$(i,j) \in \mathcal{E}_{\text{in}}$

$$\phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) = \lambda(1 - 2|s_{ij} - s_{ji}|) + (2 s_{ij} s_{ji} - 1)\left[\boldsymbol{\eta}^T |\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T (\mathbf{x}_i \mathbf{x}_j)\right]. \tag{6}$$

Parameters $= [\boldsymbol{\beta}, \mathbf{a}]$

Node and Pairwise Potentials for MFA

Node Potential: For each node $i \in \mathcal{V}$

$$\phi_i(\mathbf{s}_i) = \sum_{v=0}^{D} \beta_v \mathbb{I}\Big[\sum_j s_{ij} = v\Big] + \mathbf{a}^T \mathbf{x}_i \sum_j s_{ij}, \tag{5}$$

Pairwise Potential: For each edge, $(i,j) \in \mathcal{E}_{\text{in}}$

$$\phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) = \lambda(1 - 2|s_{ij} - s_{ji}|) + (2 s_{ij} s_{ji} - 1)\Big[\boldsymbol{\eta}^T |\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T (\mathbf{x}_i \mathbf{x}_j)\Big]. \tag{6}$$

Parameters $= [\boldsymbol{\beta}, \mathbf{a}, \lambda]$

Node and Pairwise Potentials for MFA

Node Potential: For each node $i \in \mathcal{V}$

$$\phi_i(\mathbf{s}_i) = \sum_{v=0}^{D} \beta_v \mathbb{I}\Big[\sum_j s_{ij} = v\Big] + \mathbf{a}^T \mathbf{x}_i \sum_j s_{ij}, \tag{5}$$

Pairwise Potential: For each edge, $(i, j) \in \mathcal{E}_{\text{in}}$

$$\phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) = \lambda\big(1 - 2|s_{ij} - s_{ji}|\big) + (2 s_{ij} s_{ji} - 1)\Big[\boldsymbol{\eta}^T |\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T (\mathbf{x}_i \mathbf{x}_j)\Big]. \tag{6}$$

Parameters $= [\boldsymbol{\beta}, \mathbf{a}, \lambda, \boldsymbol{\eta}, \boldsymbol{\nu}]$

Approximate posterior density with a simpler one

Approximate posterior density with a simpler one

Mean-Field Factorisation: $q(\mathbf{S}) \in \mathcal{Q}$

$$q(\mathbf{S}) = \prod_{i=1}^{N} \prod_{j=1}^{N} q_{ij}(s_{ij}), \tag{5}$$

Implication: Node connectivities are independent.

Approximate posterior density with a simpler one

Mean-Field Factorisation: $q(\mathbf{S}) \in \mathcal{Q}$

$$q(\mathbf{S}) = \prod_{i=1}^{N} \prod_{j=1}^{N} q_{ij}(s_{ij}), \tag{5}$$

Implication: Node connectivities are independent.

Variational Inference to approximate $p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\text{in}})$

$$p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\text{in}}) \approx q(\mathbf{S}) \tag{6}$$

Approximate posterior density with a simpler one

Mean-Field Factorisation: $q(\mathbf{S}) \in \mathcal{Q}$

$$q(\mathbf{S}) = \prod_{\mathrm{i}=1}^{N} \prod_{\mathrm{j}=1}^{N} q_{ij}(s_{ij}), \tag{5}$$

Implication: Node connectivities are independent.

Variational Inference to approximate $p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\mathrm{in}})$

$$p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\mathrm{in}}) \approx q(\mathbf{S}) \tag{6}$$

Minimize KL Divergence $\equiv$ Maximize Evidence Lower Bound (ELBO)

$$\mathrm{ELBO}(q) = -\mathrm{KLD}(q(\mathbf{S})||p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\mathrm{in}})\Big] + \ln Z \tag{7}$$

Maximising ELBO wrt $q_{ij}(s_{ij})$ yields MFA Iterations

MFA Iterations

$$\alpha_{kl}^{(t+1)} = q_{kl}^{(t+1)}(s_{kl} == 1) = \frac{1}{1 + \exp^{-\gamma_{kl}}}$$

$\forall\ k = \{1 \dots N\},\ l \in \mathcal{N}_k$

$\boldsymbol{\alpha}$: Global connectivity prediction

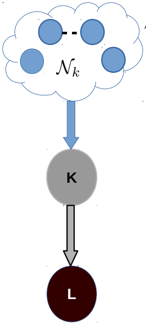
Maximising ELBO wrt $q_{ij}(s_{ij})$ yields MFA Iterations

MFA Iterations

$$\alpha_{kl}^{(t+1)} = q_{kl}^{(t+1)}(s_{kl} == 1) = \frac{1}{1+\exp^{-\gamma_{kl}}}$$

$\forall\ k = \{1 \dots N\},\ l \in \mathcal{N}_k$

$\boldsymbol{\alpha}$: Global connectivity prediction



$$\gamma_{kl} = \prod_{j \in \mathcal{N}_k \setminus l} (1-\alpha_{kj}^{(t)}) \Big\{ \sum_{m \in \mathcal{N}_k \setminus l} \frac{\alpha_{km}^{(t)}}{(1-\alpha_{km}^{(t)})} \Big[(\beta_2 - \beta_1) - \beta_2 \sum_{n \in \mathcal{N}_k \setminus l,m} \frac{\alpha_{kn}^{(t)}}{(1-\alpha_{kn}^{(t)})} \Big] + (\beta_1 - \beta_0) \Big\} + \mathbf{a}^T \mathbf{x}_i + (4\alpha_{lk}^{(t)} - 2)\lambda + 2\alpha_{lk}^{(t)} \left(\boldsymbol{\eta}^T |\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T (\mathbf{x}_i \mathbf{x}_j) \right)$$

$$\alpha_{kl}^{(t+1)} = \sigma(\gamma_{kl}) = \frac{1}{1+\exp^{-\gamma_{kl}}}$$

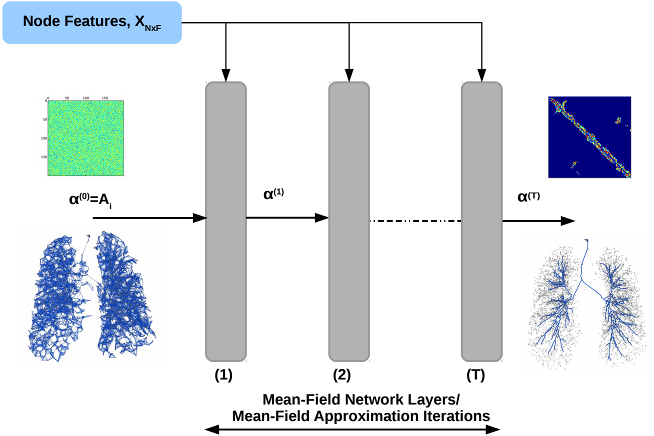
Note: MFA iterations resemble feed-forward operations in neural nets

MFA as Mean-Field Networks

- $T-$iterations as a $T-$layered network

MFA as Mean-Field Networks

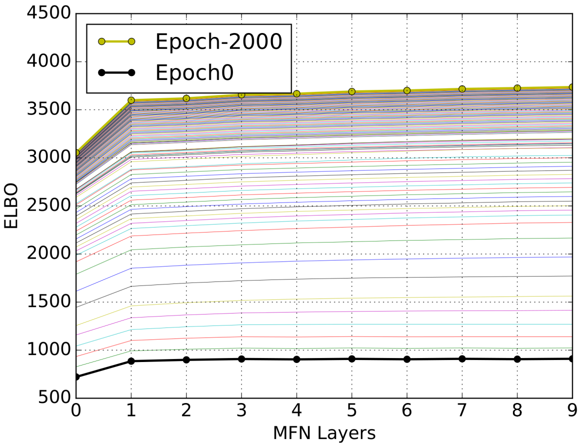
- $T-$iterations as a $T-$layered network
- Gradient descent to learn model parameters: $\mathcal{L}(\alpha, \mathbf{A}_r)$



Experiments

- Same set-up as with Bayesian smoothing
- Pretraining dataset used to tune hyperparameters
- Eight fold cross validation

Increasing ELBO $\Longrightarrow$ Better approximation

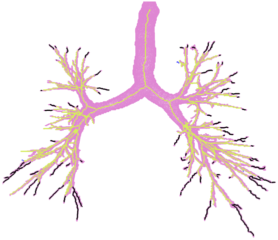


Performance comparison

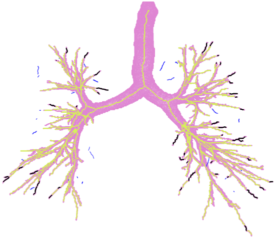
	d_{FP}(mm)	d_{FN}(mm)	d_{err} (mm)	TL(%)	FPR(%)
Vox+RG	3.624 ± 0.776	5.155 ± 0.580	4.389 ± 0.441	79.6 ± 7.2	5.0 ± 3.9
BS+RG	3.921 ± 0.612	4.218 ± 0.334	4.069 ± 0.476	82.3 ± 6.1	8.7 ± 3.4
MFN	3.599 ± 0.583	3.491 ± 0.295	3.595 ± 0.321	83.1 ± 6.7	8.6 ± 5.3

- $d_{FP} \equiv$ Specificity
- $d_{FN} \equiv$ Sensitivity
- Average centerline distance: d_{err}
- Percentage of tree length (TL)
- False positive rate (FPR)

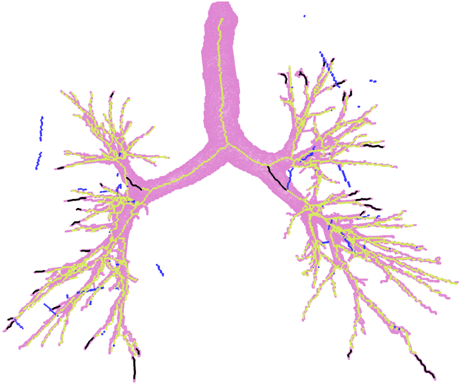
Visualisation of extracted airways



Vox+RG



BS+RG



MFN

Legend: Reference (pink), True Positive (Yellow), False Negative (Black), False Positive (Blue)

Summary

- Airway extraction as graph refinement
- Novel use of Mean-Field Approximation
- Proposed expressive node and pairwise potentials
- Mean-Field Network interpretation
- Few parameters (46 scalar weights)
- Easy to optimise using gradient descent

– Might not generalise across applications

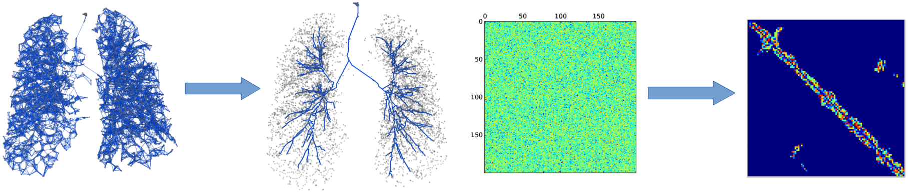
– Hand-crafting potentials is cumbersome

Graph Neural Networks

Graph Neural Networks

- Neural nets with graph input
- Step towards non-Euclidean (geometric) Deep Learning
- Generalisation of message passing algorithms
- Complex task-specific messages can be learnt
- End-to-end trainable inference systems

GNN based Graph Refinement



- Graph refinement task: $f : \mathcal{G}_{\text{in}} \rightarrow \mathcal{G}$
- GNN based encoder-decoder pair
- Encoder comprises stacks of GNNs; Message passing between nodes
- Joint training of encoder-decoder pair to learn useful embeddings
- Simple decoder predicts graph connectivity

GNN Model for Graph Refinement

GNN Model for Graph Refinement

Consider node j with neighbours $\mathcal{N}_j$,

$$\text{Node Embedding:} \quad \mathbf{h}_j^1 = g_n(\mathbf{x}_j) \tag{8}$$

$$\text{N2E mapping:} \quad \mathbf{h}_{(i,j)}^1 = g_{n2e}([\mathbf{h}_i^1, \mathbf{h}_j^1]) \tag{9}$$

$$\text{E2E mapping:} \quad \mathbf{h}_j^2 = g_{e2n}(\sum \mathbf{h}_{(i,j)}^1]) \quad \forall i \in \mathcal{N}_j \tag{10}$$

$$\text{N2E mapping:} \quad \mathbf{h}_{(i,j)}^2 = g_{n2e}([\mathbf{h}_i^2, \mathbf{h}_j^2]) \tag{11}$$

$$\textbf{Decoder:} \quad \alpha_{ij} = \sigma(g_{dec}(\mathbf{h}_{(i,j)}^2)) \tag{12}$$

$g_{...}(\cdot)$ are MLPs, g_{dec} is MLP with 1 output channel

GNN Model for Graph Refinement

Consider node j with neighbours $\mathcal{N}_j$,

$$
\begin{aligned}
\text{Node Embedding:} \quad & \mathbf{h}_j^1 &= \quad & g_n(\mathbf{x}_j) && (8)\\
\text{N2E mapping:} \quad & \mathbf{h}_{(i,j)}^1 &= \quad & g_{n2e}([\mathbf{h}_i^1, \mathbf{h}_j^1]) && (9)\\
\text{E2E mapping:} \quad & \mathbf{h}_j^2 &= \quad & g_{e2n}(\sum \mathbf{h}_{(i,j)}^1]) \quad \forall i \in \mathcal{N}_j && (10)\\
\text{N2E mapping:} \quad & \mathbf{h}_{(i,j)}^2 &= \quad & g_{n2e}([\mathbf{h}_i^2, \mathbf{h}_j^2]) && (11)\\
\textbf{Decoder:} \quad & \alpha_{ij} &= \quad & \sigma(g_{dec}(\mathbf{h}_{(i,j)}^2)) && (12)
\end{aligned}
$$

$g_{\dots}(\cdot)$ are MLPs, g_{dec} is MLP with 1 output channel

GNN Model for Graph Refinement

Consider node j with neighbours $\mathcal{N}_j$,

$$\begin{aligned}
\text{Node Embedding:} \quad \mathbf{h}_j^1 &= g_n(\mathbf{x}_j) && (8)\\
\text{N2E mapping:} \quad \mathbf{h}_{(i,j)}^1 &= g_{n2e}([\mathbf{h}_i^1, \mathbf{h}_j^1]) && (9)\\
\text{E2N mapping:} \quad \mathbf{h}_j^2 &= g_{e2n}(\sum \mathbf{h}_{(i,j)}^1]) \quad \forall i \in \mathcal{N}_j && (10)\\
\text{N2E mapping:} \quad \mathbf{h}_{(i,j)}^2 &= g_{n2e}([\mathbf{h}_i^2, \mathbf{h}_j^2]) && (11)\\
\textbf{Decoder:} \quad \alpha_{ij} &= \sigma(g_{dec}(\mathbf{h}_{(i,j)}^2)) && (12)
\end{aligned}$$

$g_{\dots}(\cdot)$ are MLPs, g_{dec} is MLP with 1 output channel

GNN Model for Graph Refinement

Consider node j with neighbours $\mathcal{N}_j$,

$$\begin{aligned}
\text{Node Embedding:} \quad \mathbf{h}_j^1 &= g_n(\mathbf{x}_j) && (8)\\
\text{N2E mapping:} \quad \mathbf{h}_{(i,j)}^1 &= g_{n2e}([\mathbf{h}_i^1, \mathbf{h}_j^1]) && (9)\\
\text{E2N mapping:} \quad \mathbf{h}_j^2 &= g_{e2n}(\sum \mathbf{h}_{(i,j)}^1]) \quad \forall i \in \mathcal{N}_j && (10)\\
\text{N2E mapping:} \quad \mathbf{h}_{(i,j)}^2 &= g_{n2e}([\mathbf{h}_i^2, \mathbf{h}_j^2]) && (11)\\
\textbf{Decoder:} \quad \alpha_{ij} &= \sigma(g_{dec}(\mathbf{h}_{(i,j)}^2)) && (12)
\end{aligned}$$

$g_{\dots}(\cdot)$ are MLPs, g_{dec} is MLP with 1 output channel

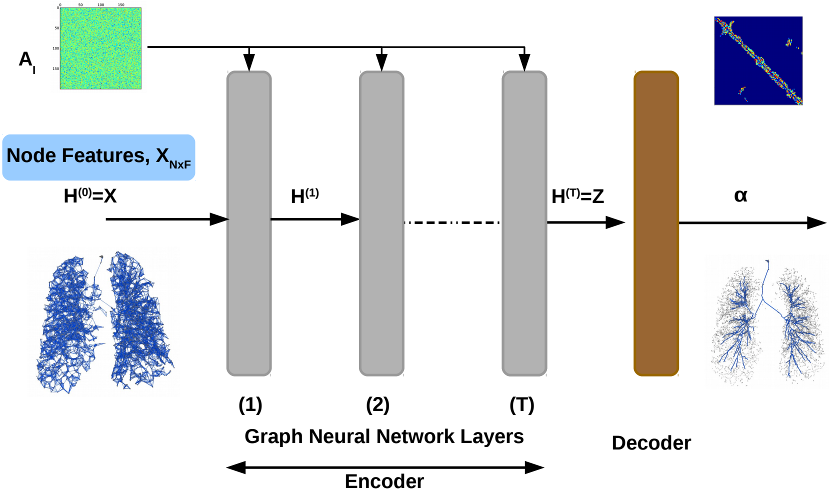
GNN Model for Graph Refinement

Consider node j with neighbours $\mathcal{N}_j$,

$$\begin{aligned}
\text{Node Embedding:} \quad \mathbf{h}_j^1 &= g_n(\mathbf{x}_j) && (8)\\
\text{N2E mapping:} \quad \mathbf{h}_{(i,j)}^1 &= g_{n2e}([\mathbf{h}_i^1, \mathbf{h}_j^1]) && (9)\\
\text{E2N mapping:} \quad \mathbf{h}_j^2 &= g_{e2n}(\sum \mathbf{h}_{(i,j)}^1]) \quad \forall i \in \mathcal{N}_j && (10)\\
\text{N2E mapping:} \quad \mathbf{h}_{(i,j)}^2 &= g_{n2e}([\mathbf{h}_i^2, \mathbf{h}_j^2]) && (11)\\
\textbf{Decoder:} \quad \alpha_{ij} &= \sigma(g_{dec}(\mathbf{h}_{(i,j)}^2)) && (12)
\end{aligned}$$

$g_{\dots}(\cdot)$ are MLPs, g_{dec} is MLP with 1 output channel

Summarising GNN Model



Experiments

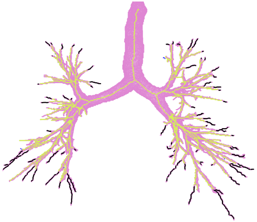
- Same set-up as with Bayesian smoothing, MFNs
- Pretraining dataset used to tune hyperparameters
- Eight fold cross validation

Performance comparison

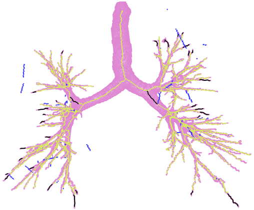
	d_{FP}(mm)	d_{FN}(mm)	d_{err} (mm)	TL(%)	FPR(%)
Vox+RG	3.624 ± 0.776	5.155 ± 0.580	4.389 ± 0.441	79.6 ± 7.2	5.0 ± 3.9
BS+RG	3.921 ± 0.612	4.218 ± 0.334	4.069 ± 0.476	82.3 ± 6.1	8.7 ± 3.4
MFN	3.599 ± 0.583	3.491 ± 0.295	3.595 ± 0.321	83.1 ± 6.7	8.6 ± 5.3
GNN	3.045 ± 0.329	2.951 ± 0.757	2.998 ± 0.399	85.3 ± 6.7	4.7 ± 3.3

- $d_{FP} \equiv$ Specificity
- $d_{FN} \equiv$ Sensitivity
- Average centerline distance: d_{err}
- Percentage of tree length (TL)
- False positive rate (FPR)

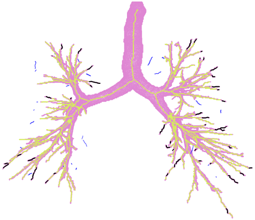
Visualisation of extracted airways



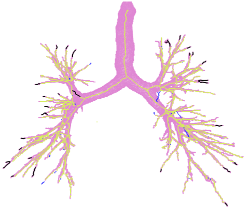
Vox+RG



MFN



BS+RG



GNN

Legend: Reference (pink), True Positive (Yellow), False Negative (Black), False Positive (Blue)

Summary

- GNN based supervised graph refinement
- Unique, inductive graph application of GNNs
- Edge embeddings used for prediction
- Competitive results with limited data
- Generalisations of MFNs
- – Disconnected trees
- – Relies on quality labelled training data

Outline

Summary of contributions

Addressed airway extraction from volumetric data with:

- Four exploratory methods
  - Modified MHT method
  - Bayesian smoothing
  - Mean-Field Networks
  - Graph Neural Networks
- Experimental validation on CT data
- Performance comparison with relevant baselines, mutual

Conclusions from the study

- Exploratory methods can extract more branches
- Graph based representations are less computationally intensive
- Using global information in local decisions is helpful
- Incorporating prior knowledge is valuable
- MFNs as structured neural networks
- GNNs as generalisations of message passing algorithms
- Bias-variance trade-off between MFNs and GNNs

Acknowledgements

- Max Welling
- Thomas Kipf
- BIGR group, Erasmus MC

Independent Research Fund Denmark (DFF)

Netherlands Organisation for Scientific Research

And of course, The Image Section!

And of course, The Image Section!