UNIVERSITY OF COPENHAGEN

### Extraction of Airways from Volumetric Data

A graph refinement view

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#### Respiratory diseases: Major cause of morbidity & mortality

#### Top 10 global causes of deaths, 2016



Source: Global Health Estimates 2016, World Health Organization, 2018



### Outline

Objective of the study

2 Data

**3** Graph Refinement Models

**4** Summary & Conclusions

**6** Supplementary material



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### Imaging based analysis of airways & challenges

Three primary steps:

1 Detection of airways

- Ø Measurement of airway morphology
- Oeriving biomarkers



Coronal view of chest CT scan



### Methods exist. Majority of them are sequential



Sequential segmentation methods

Lo, P., et.al : Extraction of airways from CT (EXACT'09). IEEE Transactions on Medical Imaging, (2012)

Juarez AG, et al. Automatic airway segmentation in chest CT using convolutional neural networks. In Image Analysis for Moving Organ, Breast, and Thoracic Images 2018.



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Sequential segmentation methods

- Susceptible to occlusions in data
- Small branches are challenging
- EXACT'09 Study
  - o Airway extraction challenge
  - o Compares 15 methods
  - o 10 use region growing!
- Quite recently some U-net based attempts on patches

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### Objective of this work

#### Extraction of airways from volumetric data

With automatic methods that:

- Are exploratory
- Use more global information in local decisions



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# Data from Danish Lung Cancer Screening Trial (DLCST)

- $\bullet$  > 10,000 Low-dose CT from 2052 subjects
- Smoker or former smoker (> 20 pack years)
- Voxels  $\sim 0.75 \times 0.75 \times 1 \ \text{mm}^3$
- 32 scans with manual annotations for evaluation
- And additional 100 with automatic segmentations for hyperparameter tuning



Pedersen, J. H., et.al : The Danish randomized lung cancer CT screening trial – Overall design and results of the prevalence round. Journal of Thoracic Oncology, (2009)

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### Graph Refinement Models

Work based on

- [1] Raghavendra Selvan, Thomas Kipf, Max Welling, Antonio GU Juarez, Jesper H. Pedersen, Jens Petersen, and Marleen de Bruijne. "Graph Refinement based airway extraction using Mean-Field Networks and Graph Neural Networks" Preprint/Medical Image Analysis (2018/2020).
- [2] Raghavendra Selvan, Max Welling, Jesper H. Pedersen, Jens Petersen, and Marleen de Bruijne. "Mean field network based graph refinement with application to airway tree extraction." 21st Conference on Medical Image Computing & Computer Assisted Intervention (MICCAI 2018), pp. 750-758, Cham. Springer International Publishing.
- [3] Raghavendra Selvan, Thomas Kipf, Max Welling, Jesper H. Pedersen, Jens Petersen, and Marleen de Bruijne. "Extraction of Airways using Graph Neural Networks." 1st Conference on Medical Imaging with Deep Learning (MIDL 2018), Amsterdam.



### Graph Refinement Model for Airway Extraction

#### High level idea

- Assume over-connected graphs with node attributes
- Optimise global connectivity, instead of qualifying individual branches



### Volumetric data to Graph data







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### Volumetric data to Graph data



- Overconnected input graph:  $\mathcal{G}_{in}$  :  $\{\mathcal{V}, \mathcal{E}_{in}\}$ , with  $|\mathcal{V}| = N$
- Node features:  $\mathbf{X} \in \mathbb{R}^{F \times N}$
- Input adjacency:  $\mathbf{A}_{in} \in \{0, 1\}^{N \times N}$



### Airway extraction as Graph Refinement task

#### Graph Refinement Model

 $f(\cdot):\mathcal{G}_{\mathsf{in}}\mapsto\mathcal{G}$ Output subgraph  $\mathcal{G}$  with  $\mathcal{E}\subset\mathcal{E}_{\mathit{in}}$ ;  $\mathbf{A}\in\{0,1\}^{N imes N}$ 







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Posterior density of interest:  $p(S|X, A_{in})$ 



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Posterior density of interest:  $p(S|X, A_{in})$ 

$$\ln p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{in}) \propto \ln p(\mathbf{S}, \mathbf{X}, \mathbf{A}_{in})$$
  
=  $\sum_{i \in \mathcal{N}} \phi_i(\mathbf{s}_i) + \sum_{(i,j) \in \mathcal{E}} \phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) - \ln Z,$ 



## Node Potential: For each node $i \in \mathcal{V}$ $\phi_i(\mathbf{s}_i) = \sum_{v=0}^{D} \beta_v \mathbb{I}\left[\sum_{j} s_{ij} = v\right] + a^T \mathbf{x}_i \sum_{j} s_{ij}, \quad (5)$

#### Pairwise Potential: For each edge, $(i,j) \in \mathcal{E}_{in}$

$$\phi_{ij}(\mathbf{s}_i, \mathbf{s}_j) = \lambda (1 - 2|s_{ij} - s_{ji}|) + (2s_{ij}s_{ji} - 1) \Big[ \boldsymbol{\eta}^T |\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T (\mathbf{x}_i \mathbf{x}_j) \Big].$$
(6)

Parameters =  $[\cdot]$ 



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Parameters =  $[\beta, a]$ 



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 $\mathsf{Parameters} = [\boldsymbol{\beta}, \mathbf{a}, \boldsymbol{\lambda}]$ 

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#### Pairwise Potential: For each edge, $(i,j) \in \mathcal{E}_{in}$

$$\phi_{ij}(\mathbf{s}_i,\mathbf{s}_j) = \lambda \big(1 - 2|s_{ij} - s_{ji}|\big) + (2s_{ij}s_{ji} - 1) \Big[\boldsymbol{\eta}^T |\mathbf{x}_i - \mathbf{x}_j| + \boldsymbol{\nu}^T (\mathbf{x}_i \mathbf{x}_j)\Big]. \quad (6)$$

$$\mathsf{Parameters} = [\boldsymbol{\beta}, \mathbf{a}, \lambda, \boldsymbol{\eta}, \boldsymbol{\nu}]$$





Mean-Field Factorisation:  $q(S) \in Q$ 

$$q(\mathbf{S}) = \prod_{\mathrm{i}=1}^{N} \prod_{\mathrm{i}=1}^{N} q_{ij}(s_{ij})$$

Implication: Node connectivities are independent.



(1)

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Variational Inference to approximate  $p(S|X, A_{in})$ 

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(1)

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Variational Inference to approximate  $p(S|X, A_{in})$ 

$$p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{in}) \approx q(\mathbf{S})$$
 (2)

(1)

(3)

Minimize KL Divergence  $\equiv$  Maximize Evidence Lower Bound (ELBO)

$$\mathsf{ELBO}(q) = -\mathsf{KLD}(q(\mathbf{S})||p(\mathbf{S}|\mathbf{X}, \mathbf{A}_{\mathsf{in}})| + \ln Z$$

### Maximising ELBO wrt $q_{ij}(s_{ij})$ yields MFA Iterations

#### MFA Iterations

$$lpha_{kl}^{(t+1)} = q_{kl}^{(t+1)}(s_{kl} == 1)$$
  
=  $\frac{1}{1 + \exp^{-\gamma_{kl}}}$ 

 $\forall k = \{1 \dots N\}, \ l \in \mathcal{N}_k$  $\boldsymbol{\alpha}$ : Global connectivity prediction



### Maximising ELBO wrt $q_{ij}(s_{ij})$ yields MFA Iterations



**Note:** MFA iterations resemble feed-forward operations in neural nets



### MFA as Mean-Field Networks

• T-iterations as a T-layered network



### MFA as Mean-Field Networks

- T-iterations as a T-layered network
- Gradient descent to learn model parameters:  $\mathcal{L}(oldsymbol{lpha}, oldsymbol{\mathsf{A}}_r)$





### Increasing ELBO $\implies$ Better approximation





### Experiments

- **Baseline:** a) Region growing on probability images b) Bayesian smoothing merged with region growing for evaluation
- Pretraining dataset used to tune hyperparameters
- Eight-fold cross validation



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- **Baseline:** a) Region growing on probability images b) Bayesian smoothing merged with region growing for evaluation
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#### • Error measures:

- Average centerline distance:  $d_{err} = (d_{FP} + d_{FN})/2$
- o  $d_{FP} \equiv$  Specificity
- o  $d_{FN} \equiv$  Sensitivity
- Percentage of tree length (TL)
- o False positive rate (FPR)



### Performance comparison

	d <sub>FP</sub> (mm)	d <sub>FN</sub> (mm)	<i>d<sub>err</sub></i> (mm)	TL(%)	FPR(%)
Vox+RG	$3.624\pm0.776$	$5.155\pm0.580$	$4.389\pm0.441$	$79.6\pm7.2$	$5.0\pm3.9$
BS+RG	$3.921\pm0.612$	$4.218\pm0.334$	$4.069\pm0.476$	$82.3\pm6.1$	$8.7\pm3.4$
MFN	$\textbf{3.599} \pm \textbf{0.583}$	$3.491 \pm 0.295$	$3.595\pm0.321$	$83.1\pm6.7$	$8.6\pm 5.3$

- $d_{FP} \equiv$  Specificity
- $d_{FN} \equiv \text{Sensitivity}$
- Average centerline distance: *d<sub>err</sub>*
- Percentage of tree length (TL)
- False positive rate (FPR)

### Visualisation of extracted airways



### Summary

- Airway extraction as graph refinement
- Novel use of Mean-Field Approximation
- Proposed expressive node and pairwise potentials
- Mean-Field Network interpretation
- Few parameters (46 scalar weights)
- Easy to optimise using gradient descent
- Might not generalise across applications
- Hand-crafting potentials is cumbersome



### Graph Neural Networks



### Graph Neural Networks

- Neural nets with graph input
- Step towards non-Euclidean (geometric) Deep Learning
- Generalisation of message passing algorithms
- Complex task-specific messages can be learnt
- End-to-end trainable inference systems



### GNN based Graph Refinement



- Graph refinement task:  $f(\cdot) : \mathcal{G}_{in} \mapsto \mathcal{G}$
- GNN based encoder-decoder pair
- Encoder comprises stacks of GNNs; Message passing between nodes
- Joint training of encoder-decoder pair to learn useful embeddings
- Simple decoder predicts graph connectivity





Consider node j with neighbours  $\mathcal{N}_j$ ,

Node Embedding:	$\mathbf{h}_{j}^{1}$	=	$g_n(\mathbf{x}_j)$	(8)
N2E mapping:	$\mathbf{h}_{(i,j)}^1$		$g_{n2e}([\mathbf{h}_i^1,\mathbf{h}_j^1])$	(9)
E2E mapping:	$\mathbf{h}_j^2$		$g_{e2n}(\sum \mathbf{h}^1_{(i,j)}]) \hspace{0.1in} orall i \in \mathcal{N}_j$	(10)
N2E mapping:	$\mathbf{h}_{(i,j)}^2$		$g_{n2e}([\mathbf{h}_i^2,\mathbf{h}_j^2])$	(11)
Decoder:	$lpha_{\it ij}$		$\sigma(g_{dec}(\mathbf{h}^2_{(i,j)}))$	(12)

 $g_{...}(\cdot)$  are MLPs,  $g_{dec}$  is MLP with 1 output channel



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### Summarising GNN Model



### Experiments

- Same set-up as with MFNs
- Pretraining dataset used to tune hyperparameters
- Eight fold cross validation



### Performance comparison

#### Table 1

Performance comparison of five methods: Region growing on probability images (Vox+RG), Bayesian smoothing merged with Vox+RG (BS+RG), UNet, MFN and GNN models. Dice similarity, centerline distances ( $d_{FP}$ ,  $d_{FN}$ ,  $d_{err}$ ), fraction of tree length detected (TL) and false positive rate (FPR) are reported based on 8–fold cross validation. Significant improvements when compared to other methods are shown in boldface. Additionally, we also report the running time to train each of the models in a single fold. Note that the MFN and GNN models require additional preprocessing that is performed only once when preparing the graphs.

	Dice(%)	$d_{FP}(mm)$	$d_{FN}(mm)$	d <sub>err</sub> (mm)	TL(%)	FPR(%)	Time (m)
Vox+RG	_	$2.937\pm1.005$	$6.762\pm2.1042$	$4.847\pm2.527$	$73.2\pm9.9$	$4.9\pm3.9$	90
BS+RG	-	$2.827\pm1.266$	$4.601\pm2.002$	$3.714 \pm 1.896$	$73.6\pm6.1$	$7.9\pm6.1$	105
UNet	-	$3.540\pm1.316$	$3.525\pm1.201$	$3.532\pm1.259$	$75.6\pm8.7$	$6.5\pm3.3$	5700
MFN	$86.5\pm2.5$	$3.608\pm1.360$	$3.116\pm0.632$	$3.362\pm1.297$	$74.5\pm6.7$	$8.6\pm5.4$	60 + 35
GNN	$84.8\pm3.3$	$2.216 \pm 0.464$	$2.878 \pm 0.505$	$2.547 \pm 0.587$	81.9 ± 7.3	$7.8\pm4.6$	60 + 12

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- Average centerline distance: *d<sub>err</sub>*
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### Summary

- GNN based supervised graph refinement
- Unique, inductive graph application of GNNs
- Edge embeddings used for prediction
- Competitive results with limited data
- Generalisations of MFNs
- Disconnected trees
- Relies on quality labelled training data



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### Conclusions from the study

- Exploratory methods can extract more branches
- Graph based representations are less computationally intensive
- Using global information in local decisions is helpful
- Incorporating prior knowledge is valuable
- MFNs as structured neural networks
- GNNs as generalisations of message passing algorithms
- Bias-variance trade-off between MFNs and GNNs



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Questions?

#### Carbontracker: Tracking and Predicting the Carbon Footprint of Training Deep Learning Models

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