Graph Refinement using GNNs

With a focus on Airway Extraction[1]

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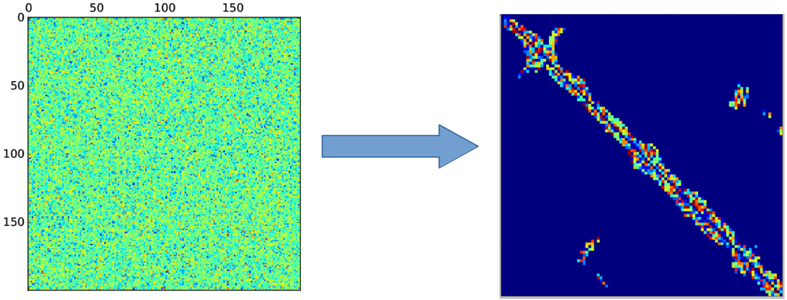
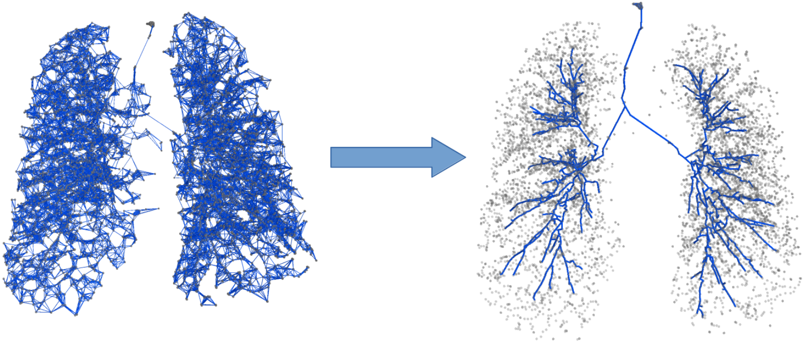
[1] Based on work in Selvan et al. "Extraction of Airways using Graph Neural Networks" (2018)

Graph Refinement from Adjacency POV



Equivalenty, Graph Refinement from Graph POV



Graph Neural Networks (GNNs)

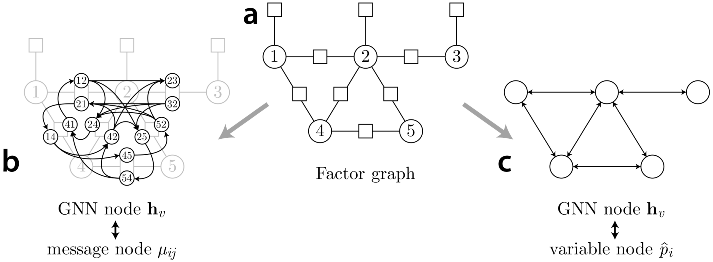


Figure: Two mappings of Factor Graphs into GNNs[2]

- Generalisation of message passing algorithms
- Arbitrarily complex messages
- End-to-end trainable inference systems

[2] Figure from Yoon et al. "Inference in probabilistic graphical models by Graph Neural Networks" (2018)

Graph Auto-Encoders (GAEs)

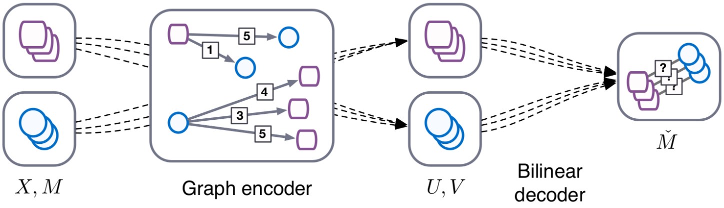


Figure: Overview of Graph Auto-Encoders[3]

- Use GNN as encoder
- Learn an embedding space using encoder
- Output predicted adjacency using decoder

[3]Figure from Berg et al. "Graph Convolutional Matrix Completion" (2017)

GAE

Encoder

$$\mathbf{H}^{(l+1)} = \sigma\left(\mathbf{H}^{(l)}\mathbf{W}_0^{(l)} + \mathbf{D}^{-1}\mathbf{A}\mathbf{H}^{(l)}\mathbf{W}_1^{(l)}\right), \quad (1)$$

where $\mathbf{A} \in \{0,1\}^{N\times N}$ is input adjacecny matrix, $\mathbf{D}$ is degree matrix : $D_{ii} = \sum_{j=1}^{N} A_{ij}$, $\mathbf{W}_0, \mathbf{W}_1 \in \mathbb{R}^{E\times E}$ are GNN weights, $\mathbf{H}^{(l)} \in \mathbb{R}^{N\times E}$ are hidden activations, $\mathbf{H}^{(0)} = \mathbf{X}$ and $\mathbf{H}^{(L)} = \mathbf{Z}$.

GAE

Decoder

$\boldsymbol{\alpha} = g(\mathbf{Z})$:

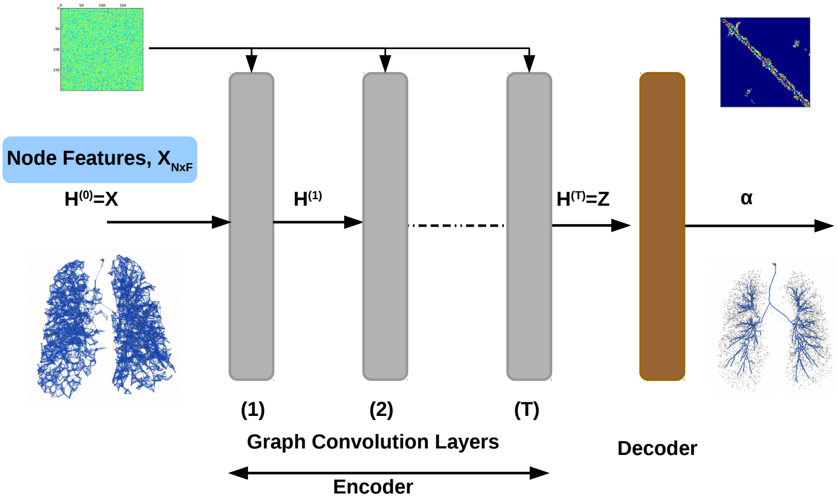
$$\alpha_{ij} = \exp -(\mathbf{z}_i - \mathbf{z}_j)^2, \quad \forall (i,j) \in \mathcal{E}. \tag{2}$$

where α_{ij} is probability of edge between nodes i, j.

Dice Loss

$$\mathcal{L}(\mathbf{A}', \boldsymbol{\alpha}) = 1 - \frac{2\sum_{i,j=1}^{N} A_{ij}\alpha_{ij}}{\sum_{i,j=1}^{N} A_{ij}^2 + \sum_{i,j=1}^{N} \alpha_{ij}^2}, \tag{3}$$

Overview of GAE for Graph Refinement

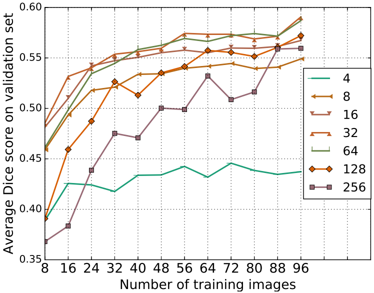


Experiments

- 32 scans with "ground-truth" for evaluation
- 100 scans with automatic reference for pre-training
- Compared with Mean-Field Networks[4]

[4]Selvan et al. "Mean field network based graph refinement with application to airway tree extraction" (2018)

Preliminary Results



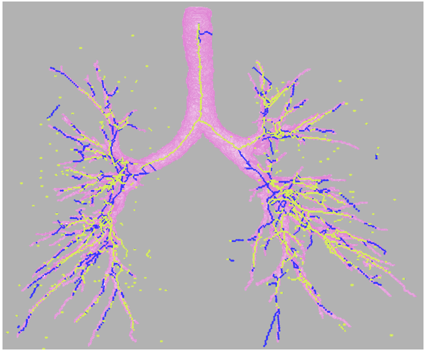


Table 1: Performance comparison using centerline distance

Method	d_{FN}(mm)	d_{FP}(mm)	d_{err} (mm)
MFN	2.571	0.835	1.703 ± 0.186
GNN	2.890	3.913	3.402 ± 0.386
GNN+MFN	2.014	3.345	2.679 ± 0.264

Conclusions

- Novel application of GNN for graph refinement
- Possible improvements with more labelled data
- Current model captures node behaviour
- Potential improvements with Edge-GNNs